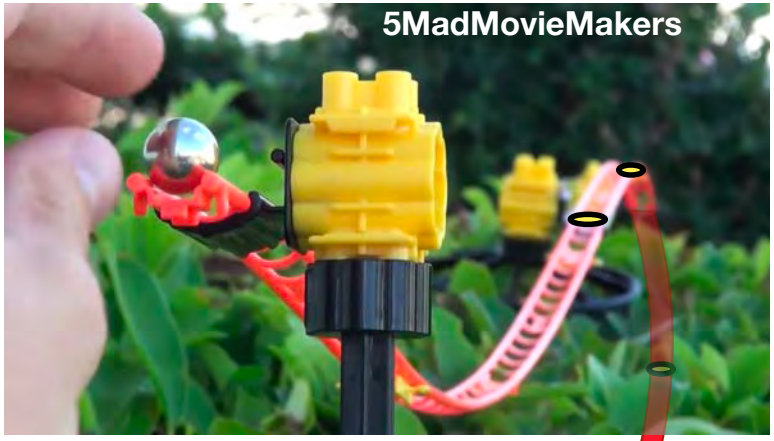
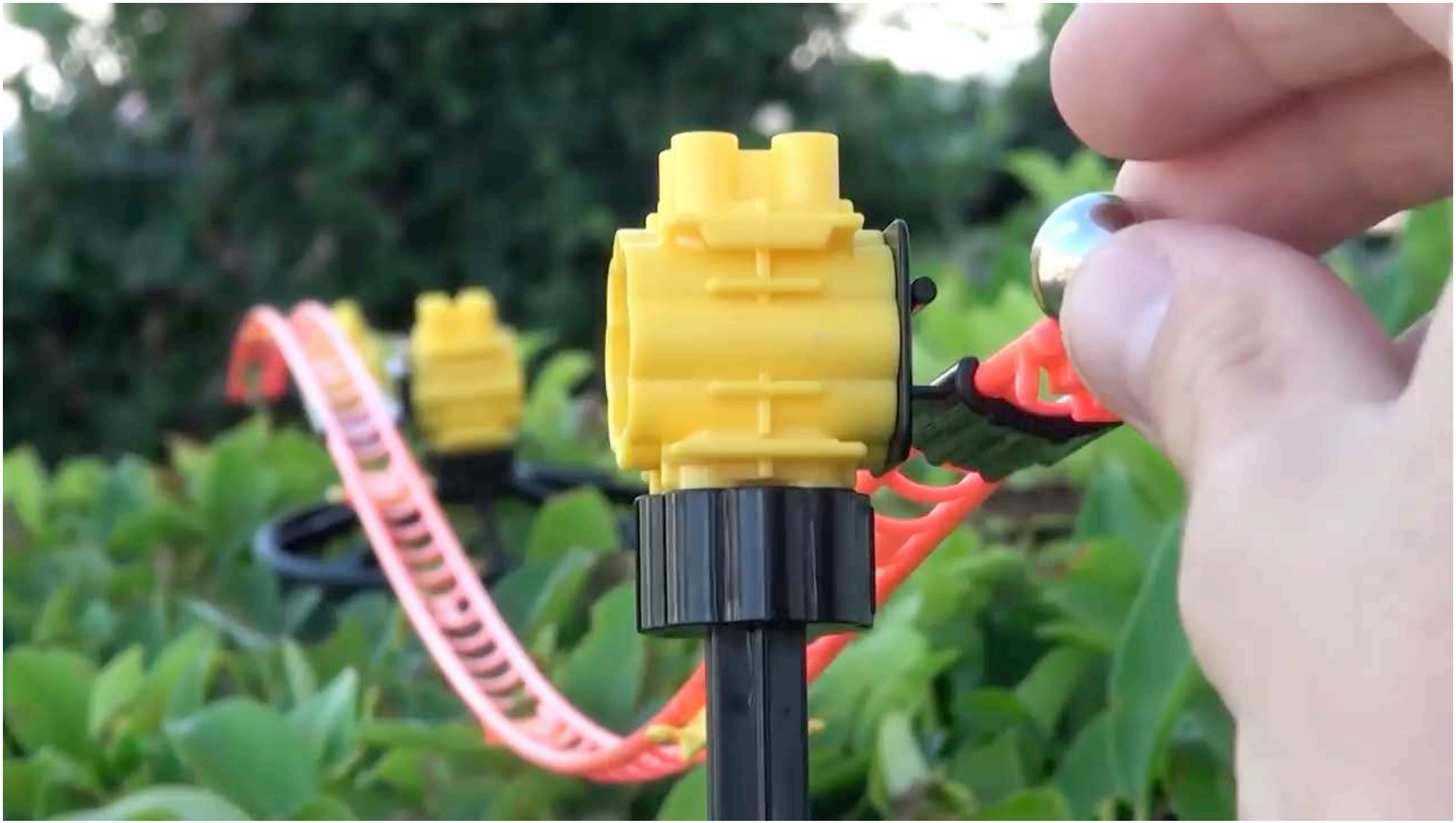


Autonomous Navigation: Local Search

Robotics 102

Introduction to AI and Programming
University of Michigan and Berea College
Fall 2021



https://www.youtube.com/watch?v=MEjsmEU6EIM

https://jellesmarbleruns.com

Search

JELLE'S MARBLE RUNS
THE MARBLE MASTER

Welcome to

MARBLE SPORTS!

The new hub for marble fans around the world - Welcome to the JMR Marble Sports homepage! With superior entertainment and production quality, Jelle's Marble Runs strives to create the very best marble racing competitions in the world.

https://www.youtube.com/watch?v=ehnyyT8Kyms

https://www.youtube.com/watch?v=7D-FHaaShvM

https://www.youtube.com/watch?v=UQGAlb_hss8

Our goal

Our goal

Give you the power of autonomous navigation

Our goal

Give you the power of autonomous navigation

Our goal

Give you the power of autonomous navigation

Autonomous Navigation

Autonomous Navigation
by local search

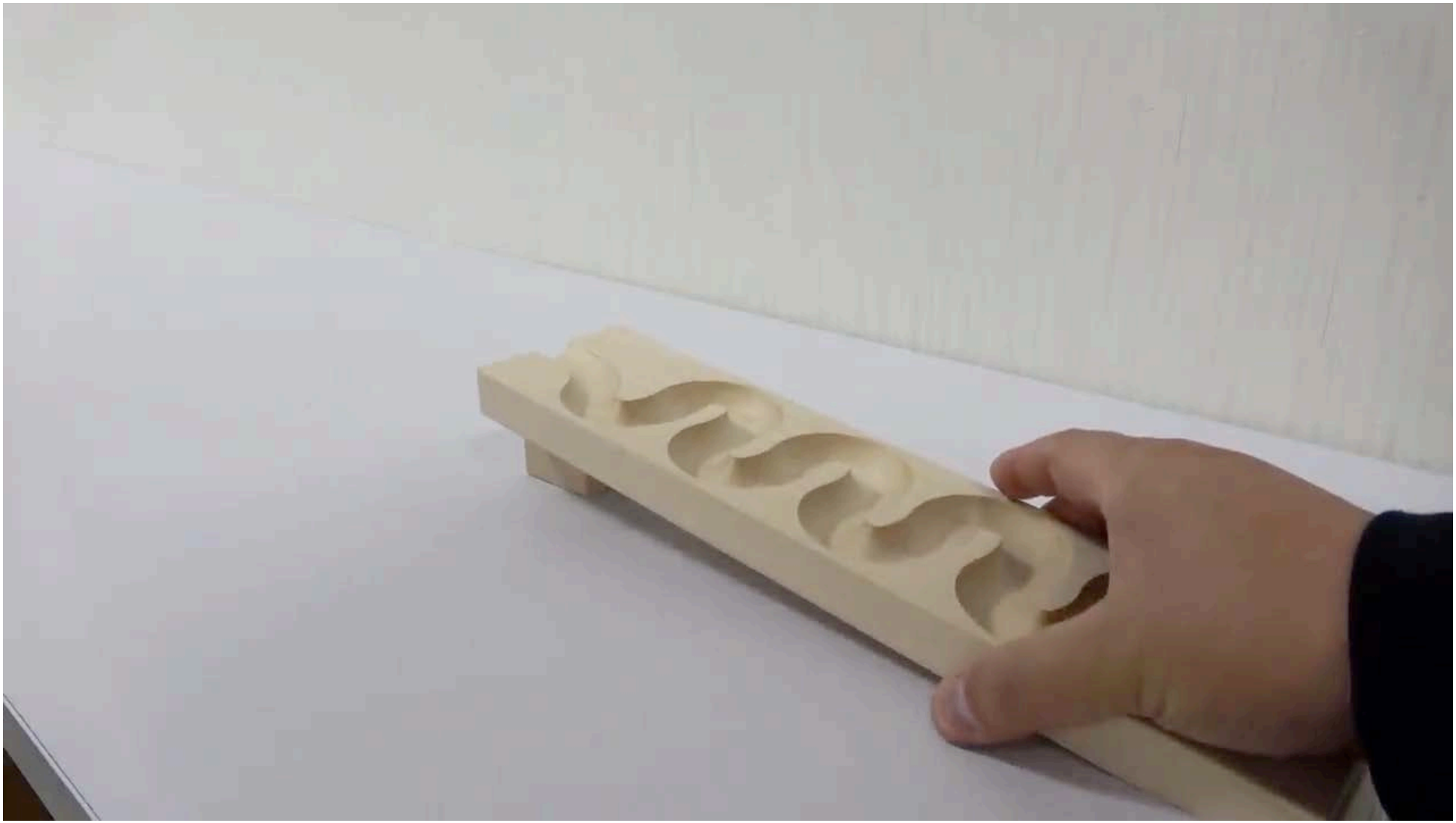
Goal location

Think of our robot's navigation
as the motion of a marble

Start location

Autonomous Navigation
by local search

https://www.youtube.com/watch?v=N_m4E0X1Unk



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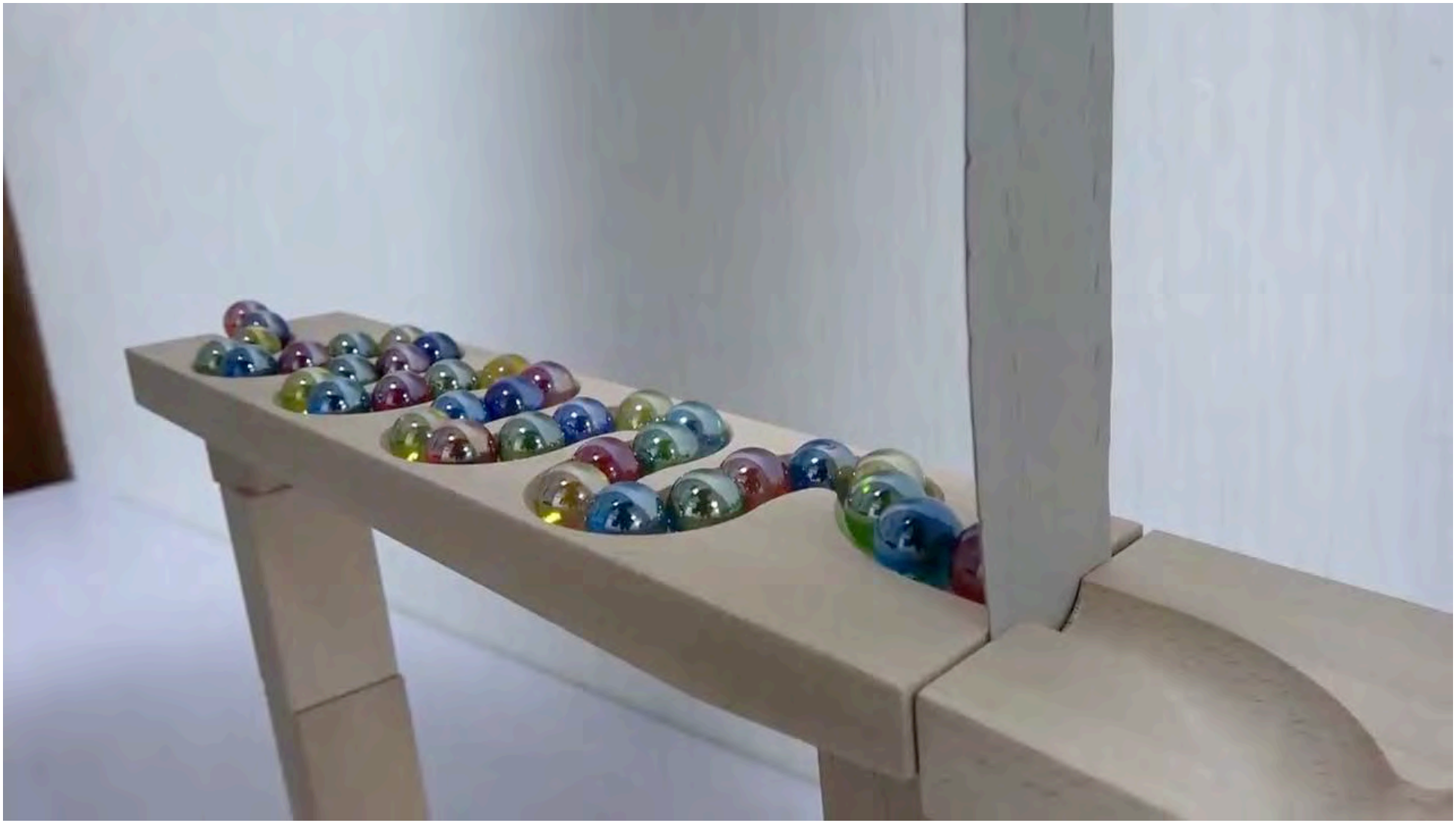


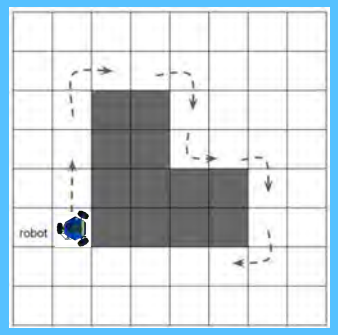
https://www.youtube.com/watch?v=N_m4E0X1Unk

Course recap to now

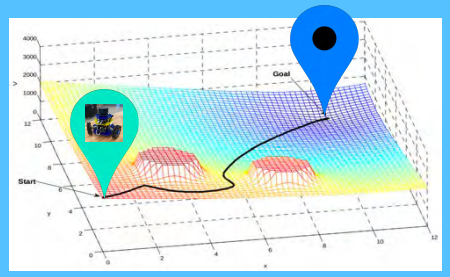
Understand foundational AI algorithms and implement them in code

C++ Programming

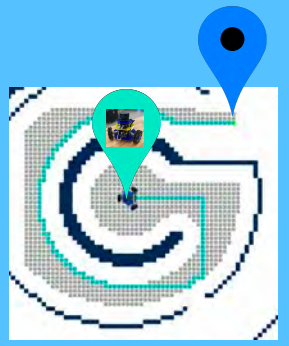
Project 0: Pocket Calculator



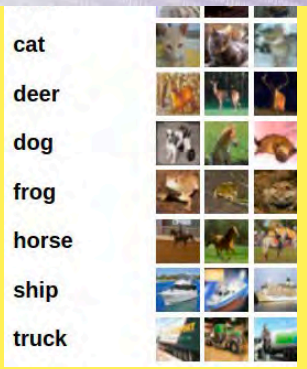
Project 1: Wall following



Project 2: Potential Fields



Project 3: A* Pathfinding



Project 4: Neural Networks

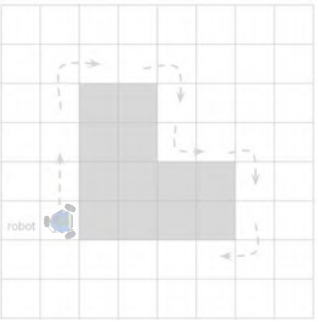
Project 0: Pocket Calculator

calculator66

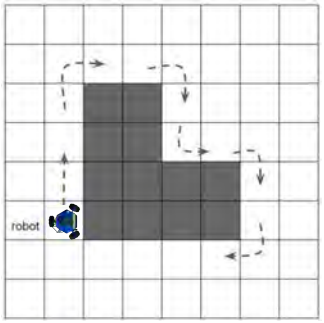
```
Please type a number and press enter: 3
Please type an operation (one of: + - * / u q): *
Please type a number and press enter: 4
(3*4) = 12
Please type an operation (one of: + - * / u q): +
Please type a number and press enter: 8
((3*4)+8) = 20
Please type an operation (one of: + - * / u q): -
Please type a number and press enter: 10
(((3*4)+8)-10) = 10
Please type an operation (one of: + - * / u q): /
Please type a number and press enter: 5
((((3*4)+8)-10)/5) = 2
Please type an operation (one of: + - * / u q): *
Please type a number and press enter: 51
(((((3*4)+8)-10)/5)*51) = 102
Please type an operation (one of: + - * / u q): q
```

- [x] Program Structure
- [x] Compile/Execute
- [x] Operators
- [x] Data Types
- [x] Variables
- [x] User Input/Output
- [x] Functions
- [x] Branching
- [x] Iterators
- [x] Vectors
- [x] Structs
- [ ] File Input/Output

Project 1: Wall following

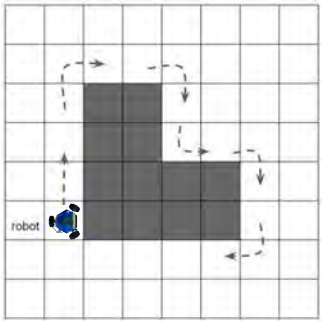


Project 1: Wall following



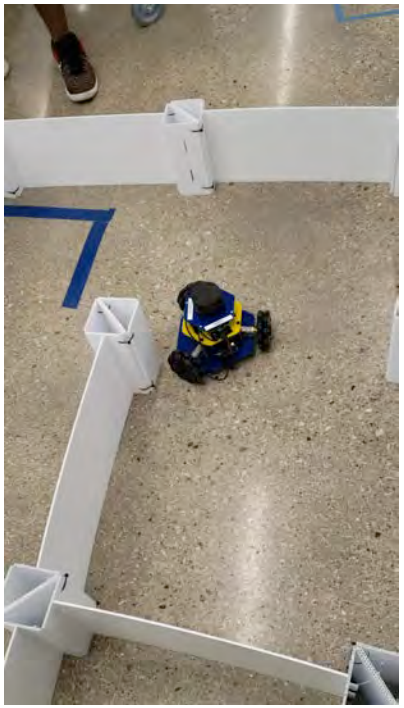
- [x] Bang-Bang Control
- [x] Find Minimum Distance
- [x] Convert Polar to Cartesian
- [x] Cross Product
- [x] Vector Sum
- [ ] Address noise

Project 1: Wall following

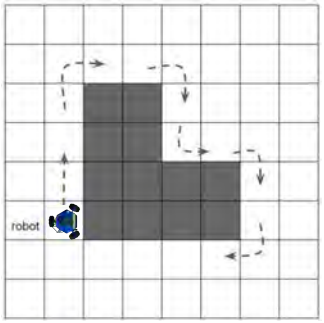


- [x] Bang-Bang Control
- [x] Find Minimum Distance
- [x] Convert Polar to Cartesian
- [x] Cross Product
- [x] Vector Sum
- [ ] Address noise

My team keeps encountering issues while rounding corners. The thought is that it's rounding the corners too wide and then no longer sensing the corner as the nearest point. We tried messing with the margins to no avail. Any thoughts?



Project 1: Wall following



- [x] Bang-Bang Control
- [x] Find Minimum Distance
- [x] Convert Polar to Cartesian
- [x] Cross Product
- [x] Vector Sum
- [ ] Address noise

My team keeps encountering issues while rounding corners. The thought is that it's rounding the corners too wide and then no longer sensing the corner as the nearest point. We tried messing with the margins to no avail. Any thoughts?

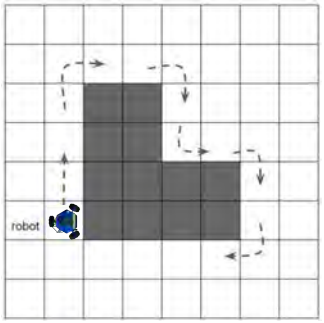
Noise is just a part of the real world

Filtering offers one way to deal with noise

Noise in this case due to outlier reading

Keep running average of robot direction

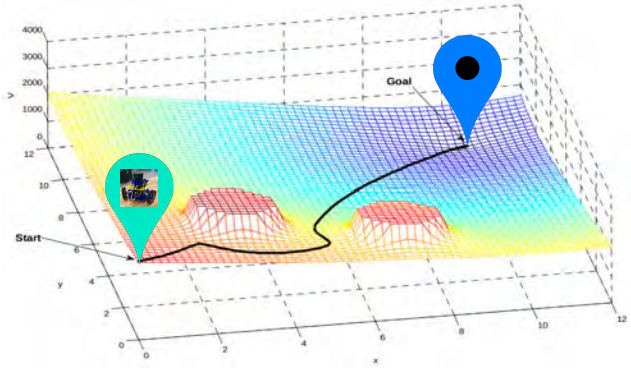
Project 1: Wall following



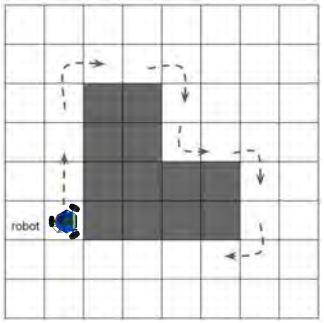
- [x] Bang-Bang Control
- [x] Find Minimum in Vector
- [x] Convert Polar to Cartesian
- [x] Cross Product
- [x] Vector Sum
- [ ] Address noise

Project 2: Potential Fields

Autonomous navigation to a goal location

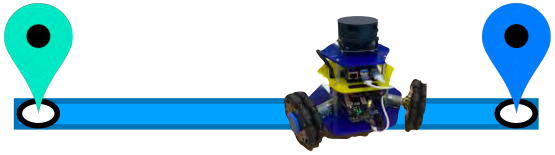


Project 1: Wall following



- [x] Bang-Bang Control
- [x] Find Minimum in Vector
- [x] Convert Polar to Cartesian
- [x] Cross Product
- [x] Vector Sum
- [ ] Address noise

Could our wall follower navigate to a goal location?



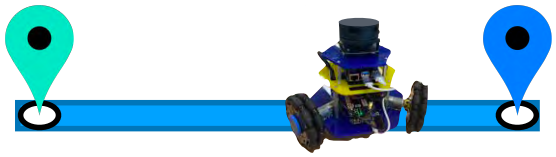
Could our wall follower navigate to a goal location?

Probably not.

Could our wall follower navigate to a goal location?

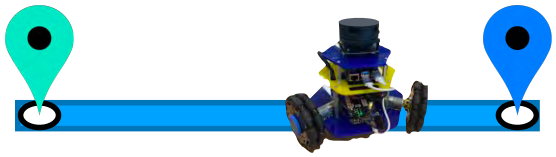
Probably not.

What options do we have for navigating our robot?



What options do we have for navigating our robot?

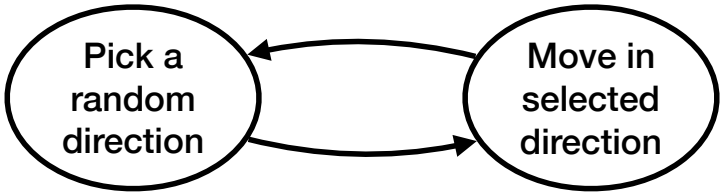
Just move randomly

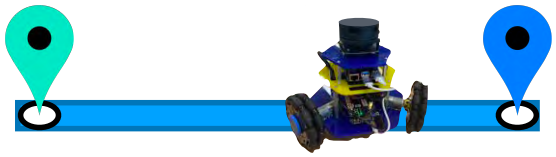


What options do we have for navigating our robot?

Just move randomly

Random walk algorithm





What options do we have for navigating our robot?

Just move randomly

Brownian motion

From Wikipedia, the free encyclopedia

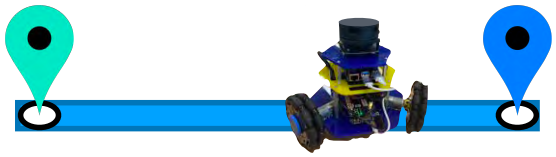
This article is about Brownian motion as a natural phenomenon. For the stochastic process, see Wiener process. For tempera internal energy, see Equipartition theorem. For the mobility model, see Random walk. For the molecular machine, see Browni

Brownian motion, or **pedesis** (from Ancient Greek: πήδησις /pɛ̌ːdɛːsis/ "leaping"), is the random motion of particles suspended i medium (a liquid or a gas).[2]

This pattern of motion typically consists of random fluctuations in a particle's position inside a fluid sub-domain, followed by a relo to another sub-domain. Each relocation is followed by more fluctuations within the new closed volume. This pattern describes a fl thermal equilibrium, defined by a given temperature. Within such a fluid, there exists no preferential direction of flow (as in transp phenomena). More specifically, the fluid's overall linear and angular momenta remain null over time. The kinetic energies of the m Brownian motions, together with those of molecular rotations and vibrations, sum up to the caloric component of a fluid's internal (the Equipartition theorem).

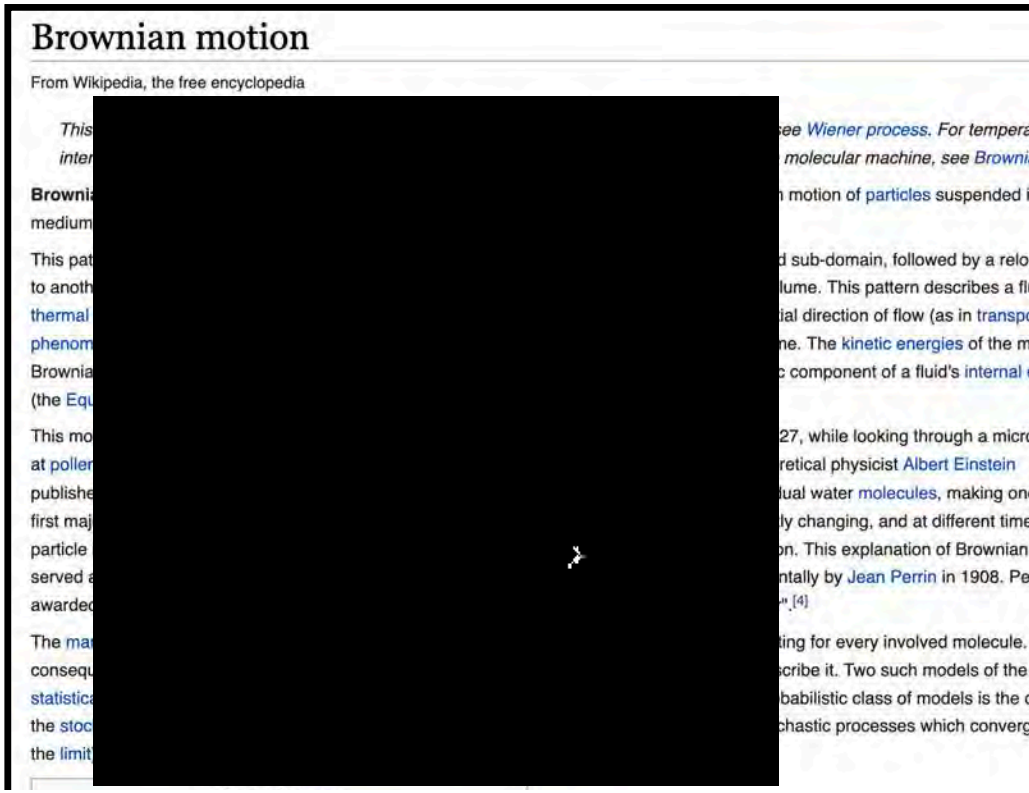
This motion is named after the botanist Robert Brown, who first described the phenomenon in 1827, while looking through a micr at pollen of the plant *Clarkia pulchella* immersed in water. In 1905, almost eighty years later, theoretical physicist Albert Einstein published a paper where he modeled the motion of the pollen particles as being moved by individual water molecules, making on first major scientific contributions.[3] The direction of the force of atomic bombardment is constantly changing, and at different time particle is hit more on one side than another, leading to the seemingly random nature of the motion. This explanation of Brownian served as convincing evidence that atoms and molecules exist and was further verified experimentally by Jean Perrin in 1908. Pe awarded the Nobel Prize in Physics in 1926 "for his work on the discontinuous structure of matter".[4]

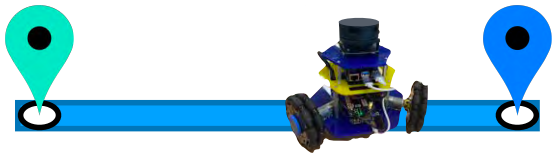
The many-body interactions that yield the Brownian pattern cannot be solved by a model accounting for every involved molecule. consequence, only probabilistic models applied to molecular populations can be employed to describe it. Two such models of the statistical mechanics, due to Einstein and Smoluchowski, are presented below. Another, pure probabilistic class of models is the c the stochastic process models. There exist sequences of both simpler and more complicated stochastic processes which converg the limit) to Brownian motion (see random walk and Donsker's theorem).[5][6]



What options do we have for navigating our robot?

Just move randomly



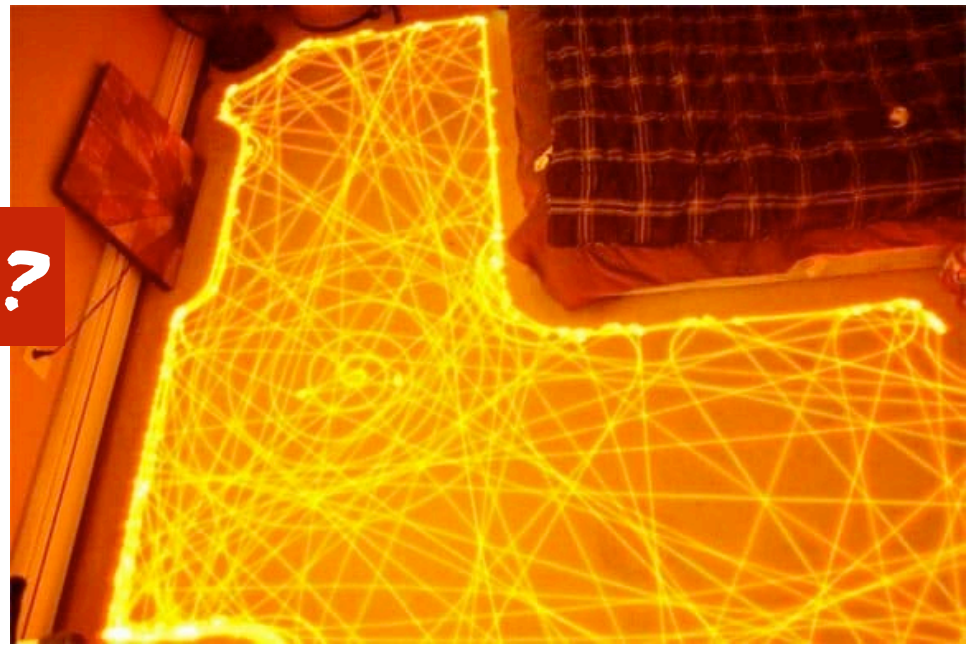


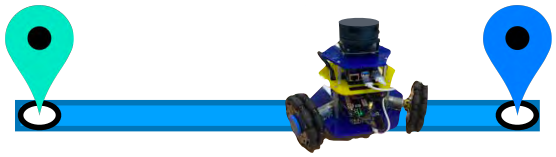
What options do we have for navigating our robot?

Just move randomly

[redacted]

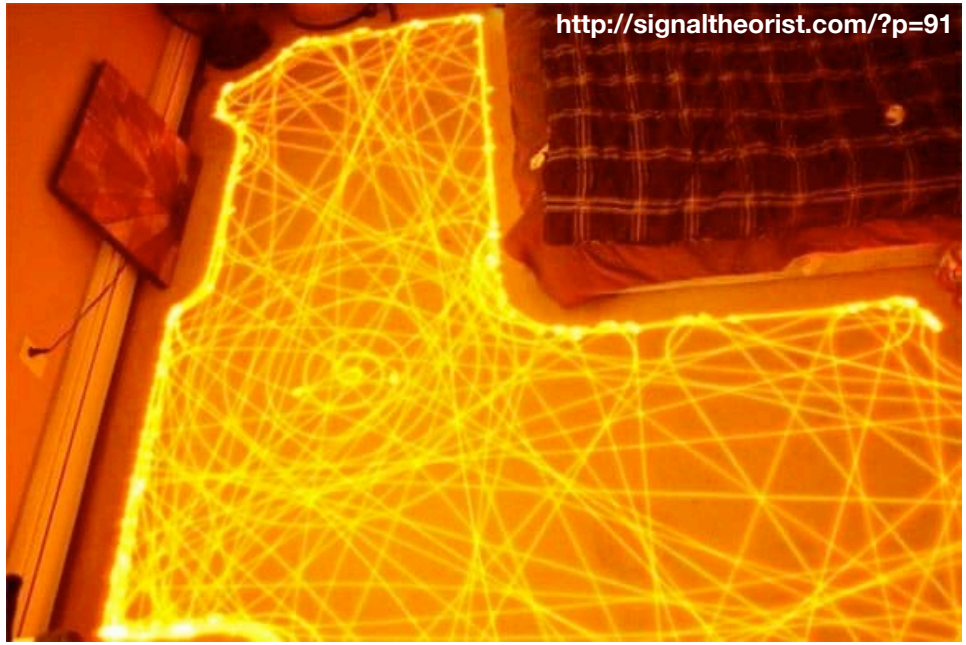
Robot that moves randomly ?



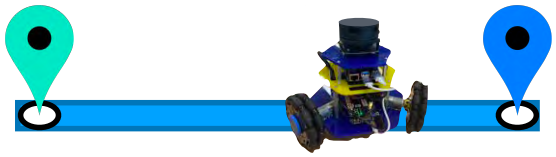


What options do we have for navigating our robot?

Just move randomly



Any benefits to random motion?

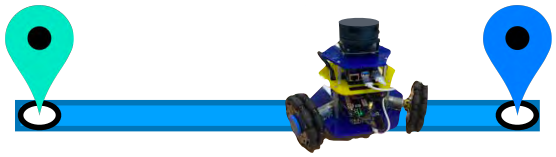


What options do we have for navigating our robot?

Just move randomly

+ Cheap
+ Simple
+ Robust
  (works in many houses)

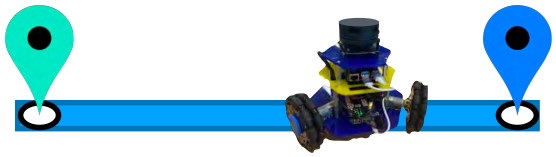
– Slow
  (will take a loooong time)



What options do we have for navigating our robot?

Just move randomly

Follow wall to goal

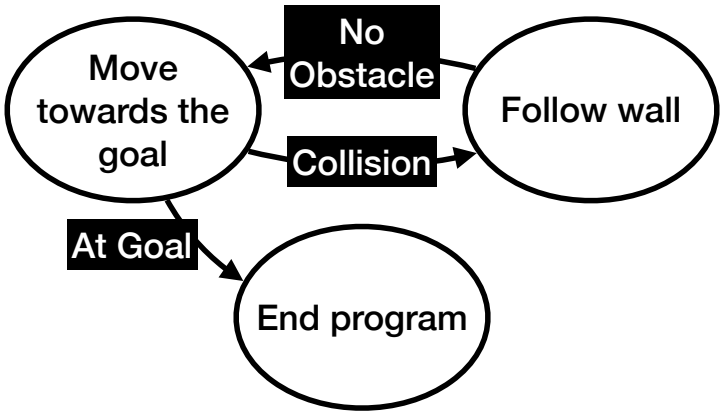


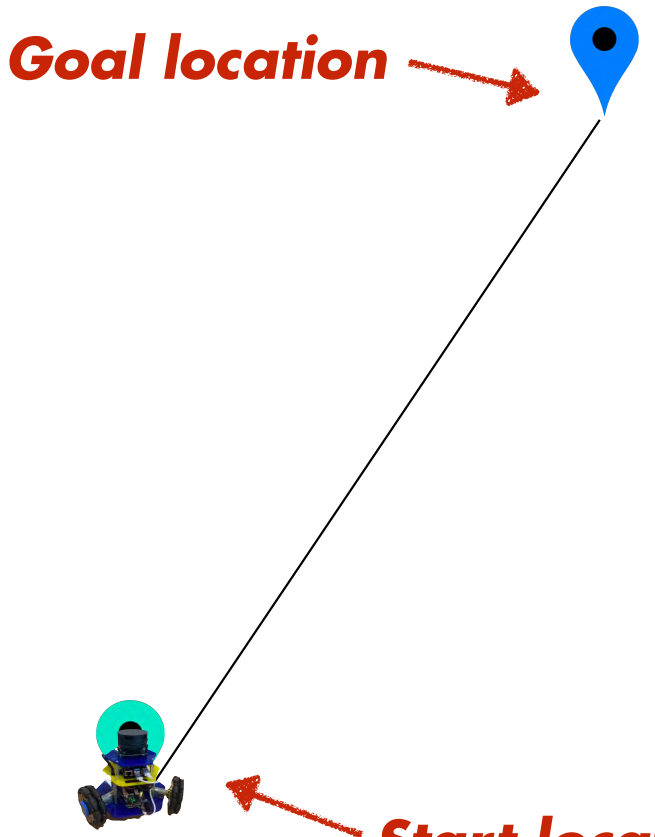
What options do we have for navigating our robot?

Just move randomly

Follow wall to goal

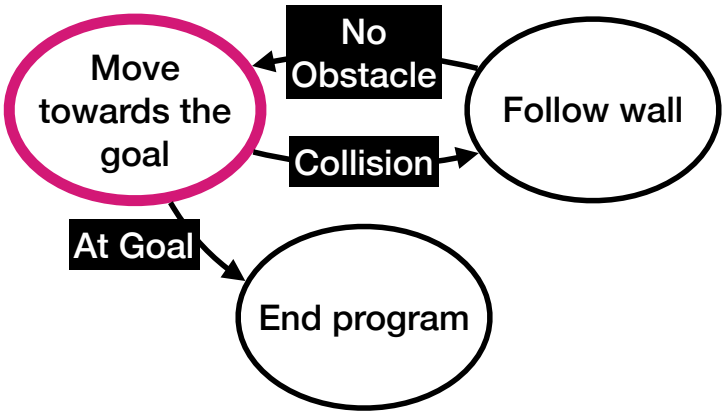
Bug algorithm

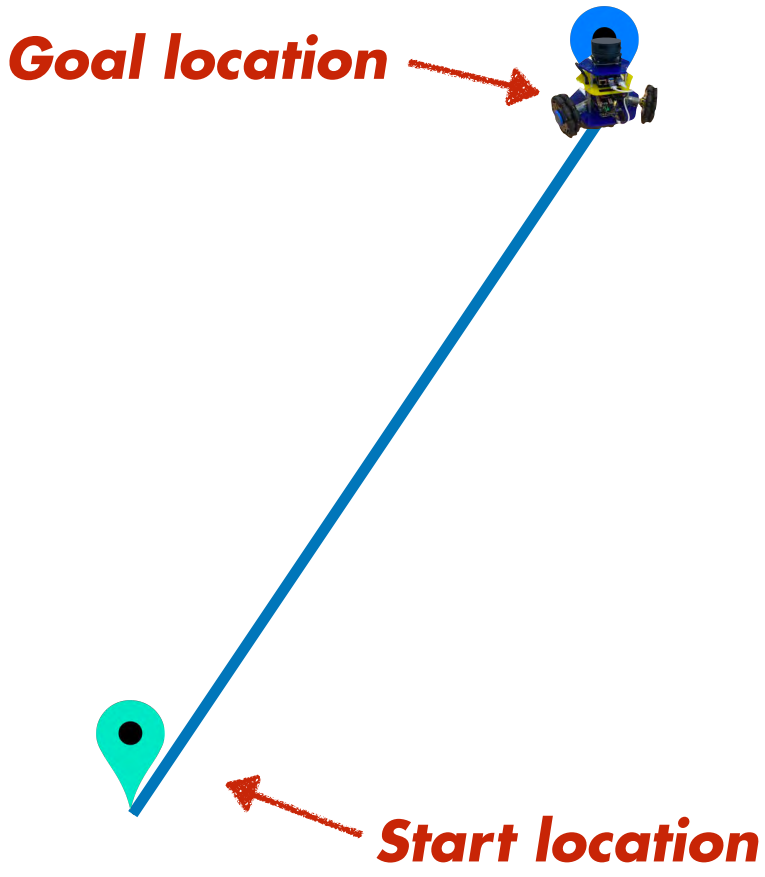




If straight line path to goal,
Just move in that direction

Bug algorithm





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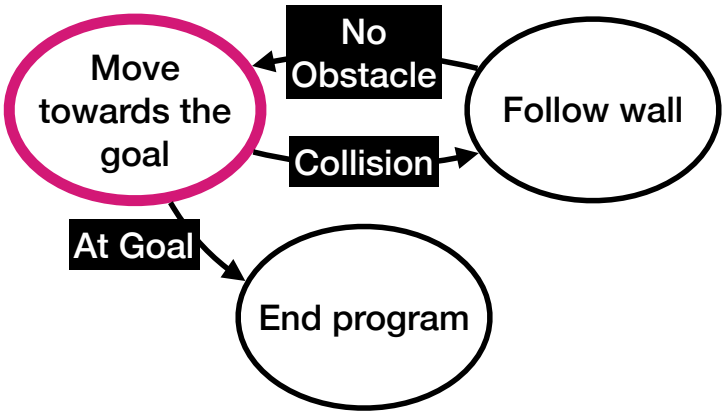
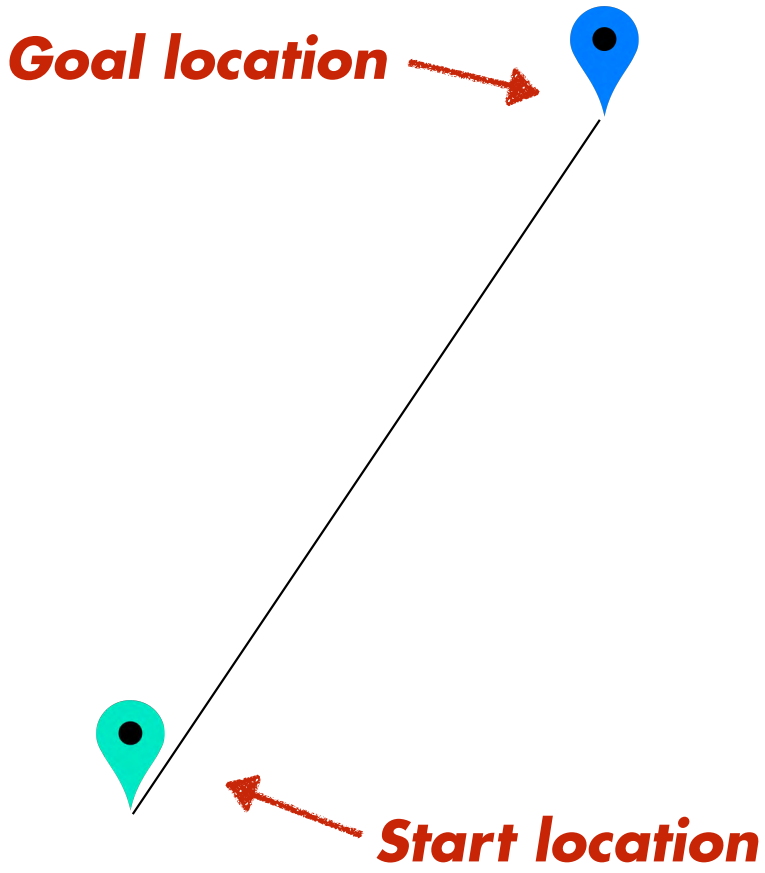
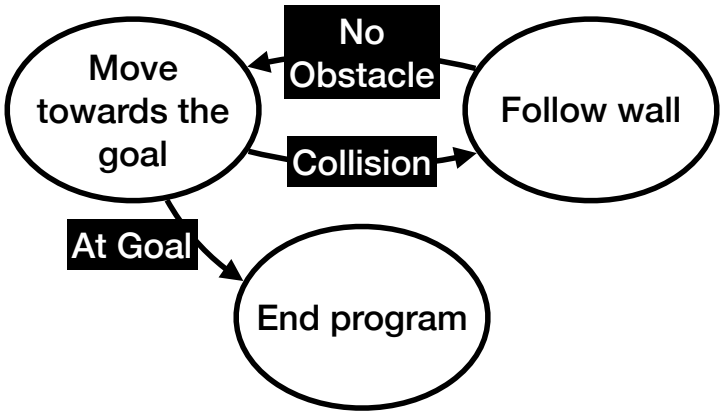
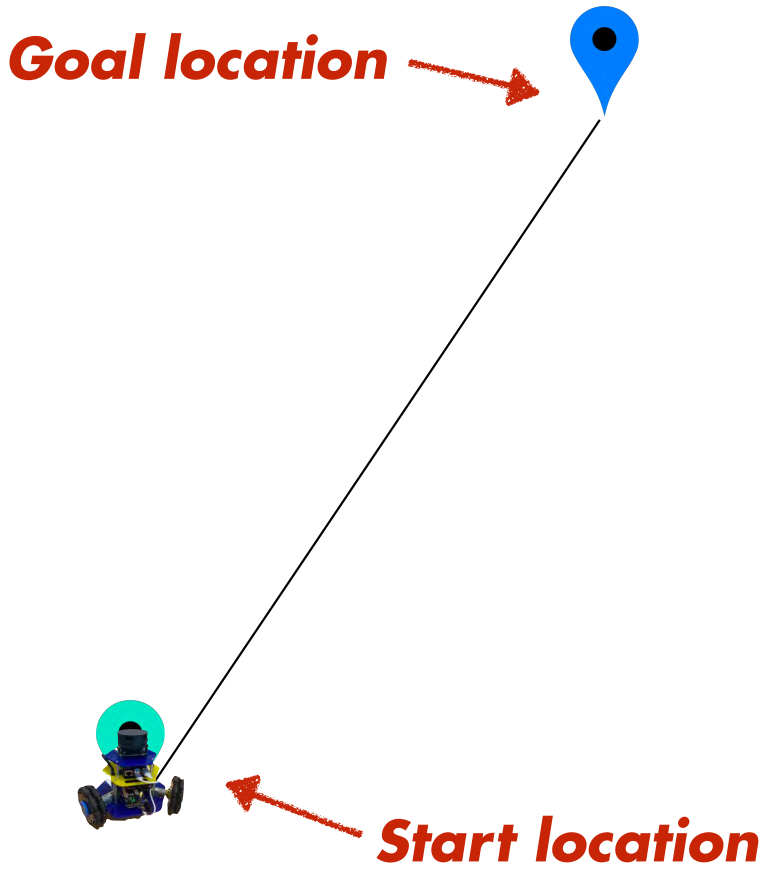




What happens if we encounter an obstacle ?

Bug algorithm





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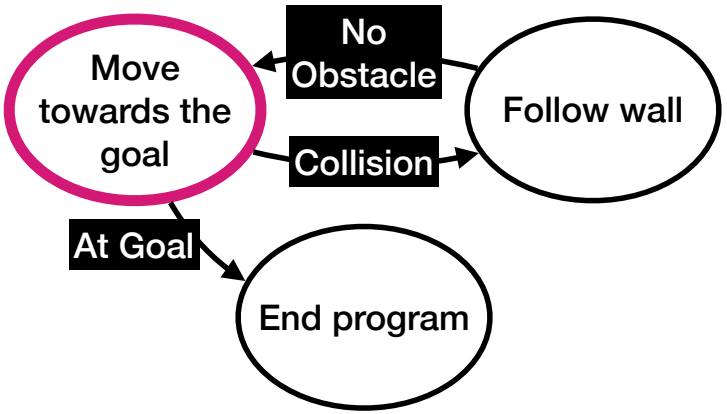

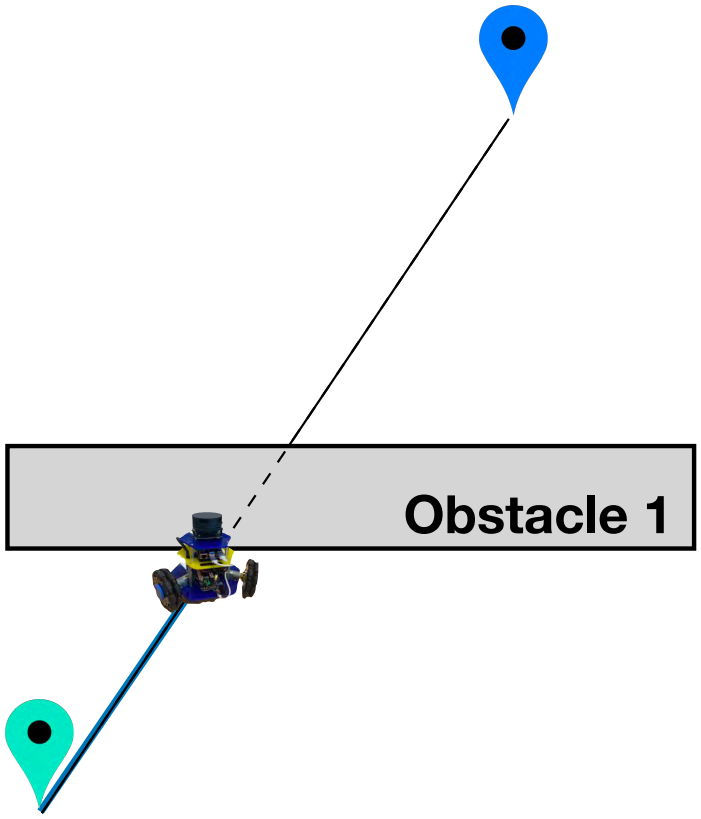

What happens if we encounter an obstacle ?

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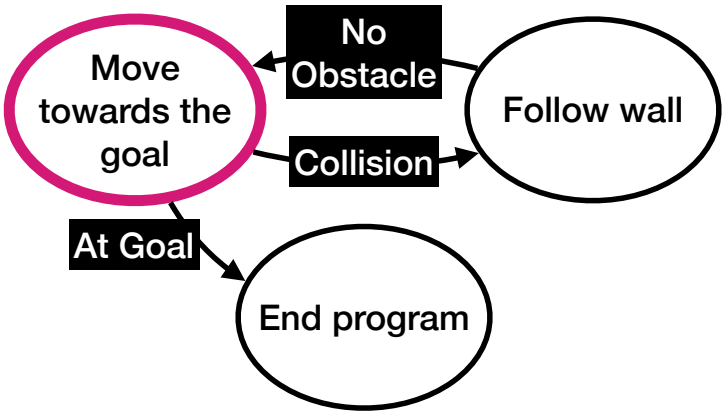
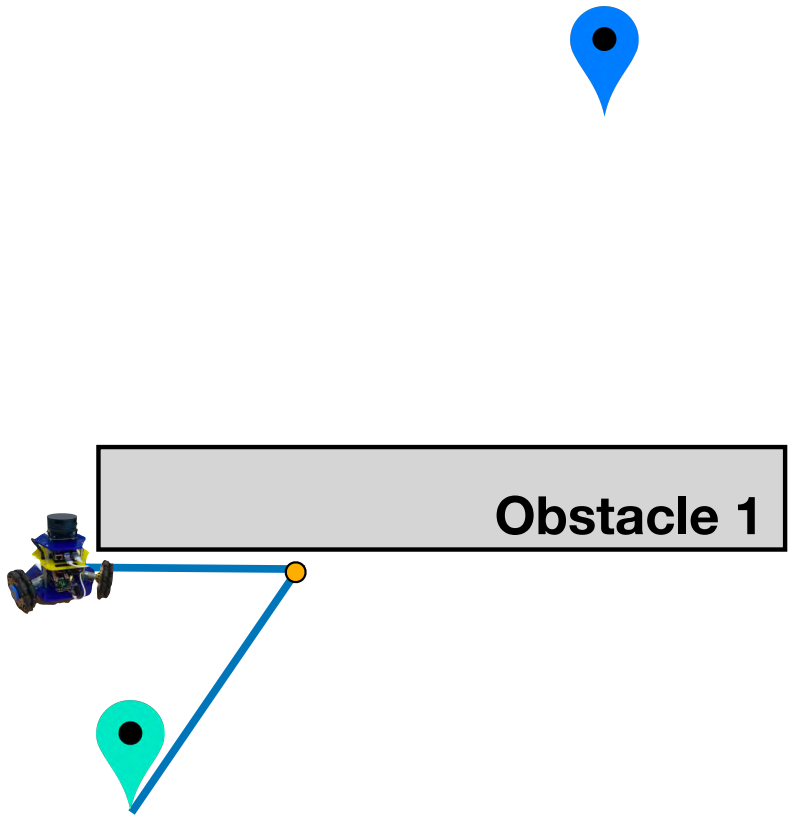
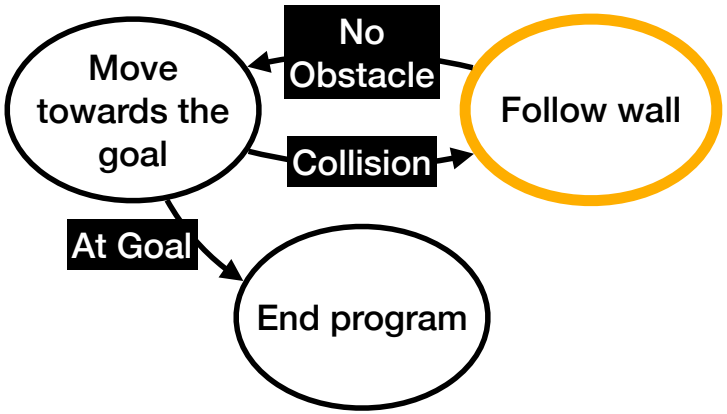
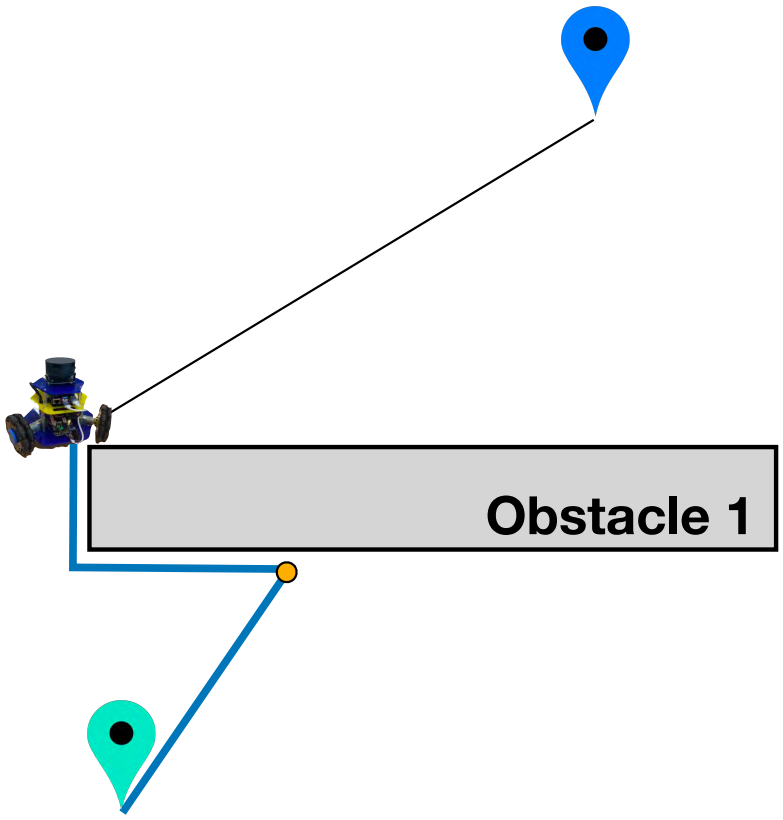




Follow wall until there is a line to the goal

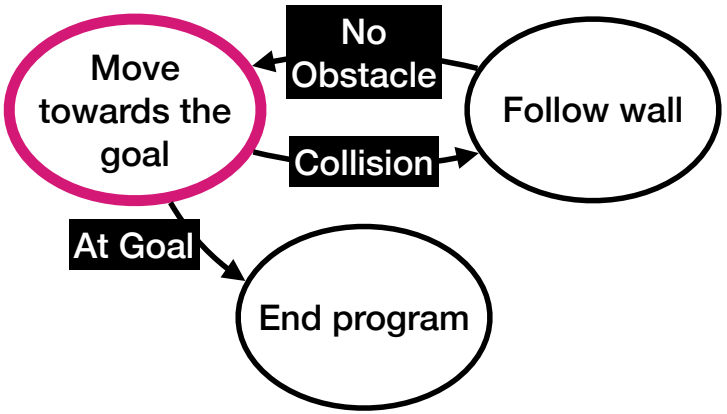
Bug algorithm

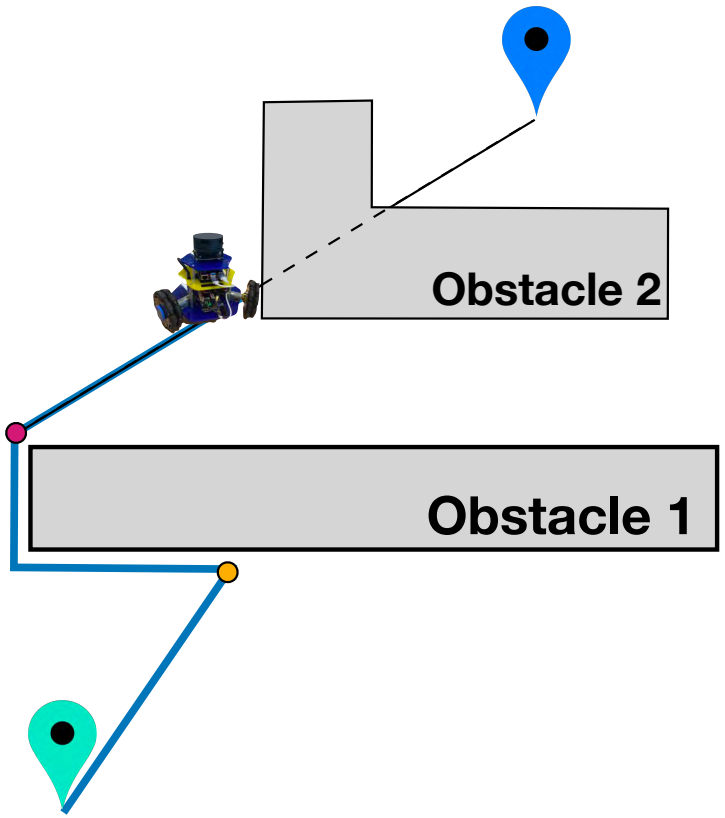




Once a line to goal is available, Move towards goal again

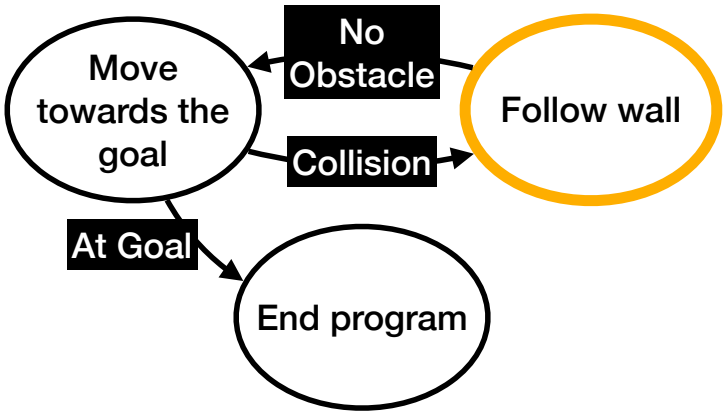
Bug algorithm

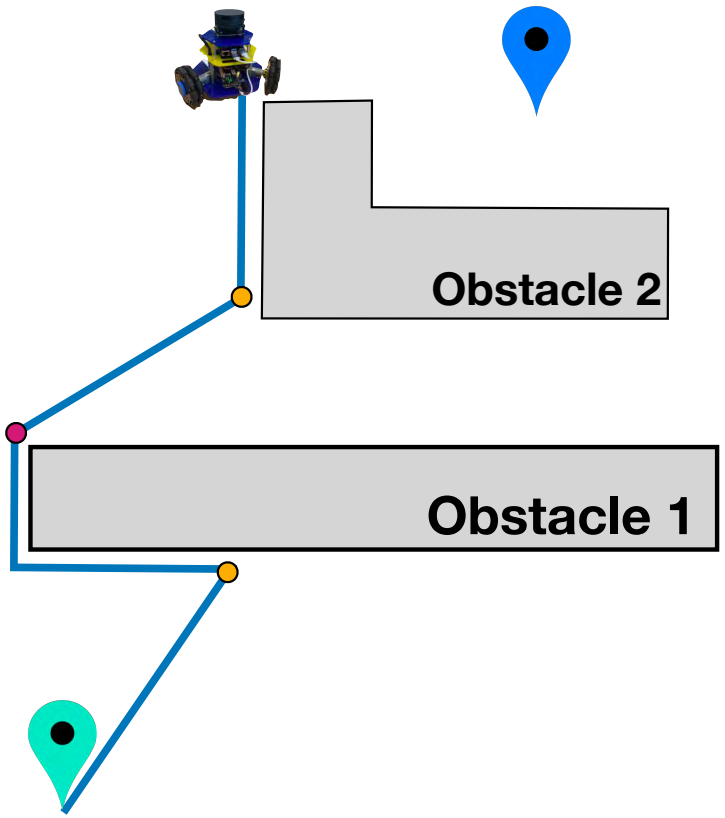




Follow wall until there is a line to the goal

Bug algorithm





Follow wall until there is a line to the goal

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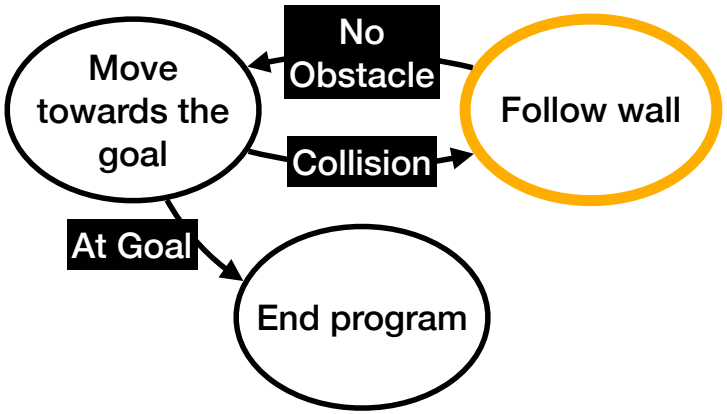
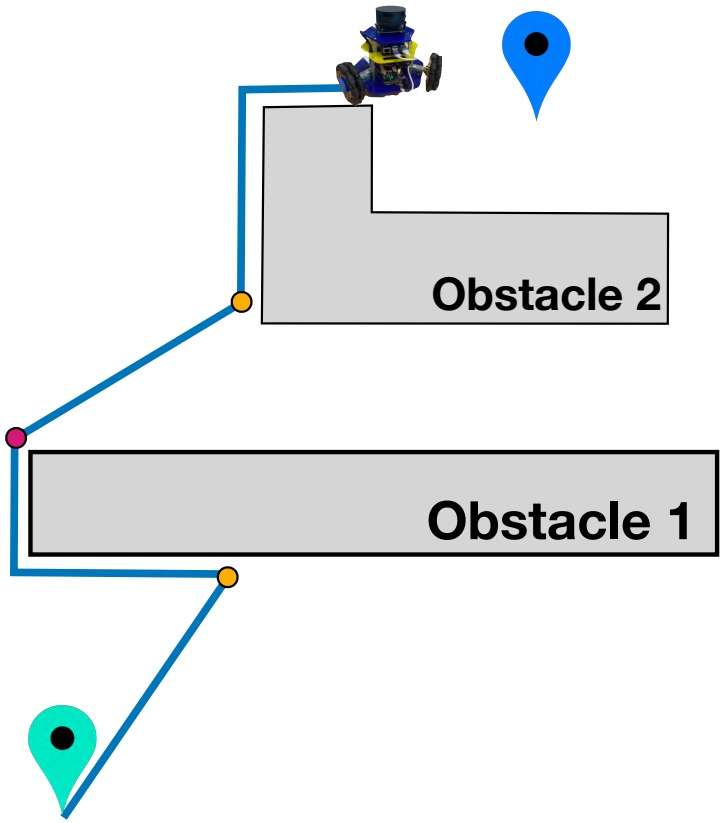
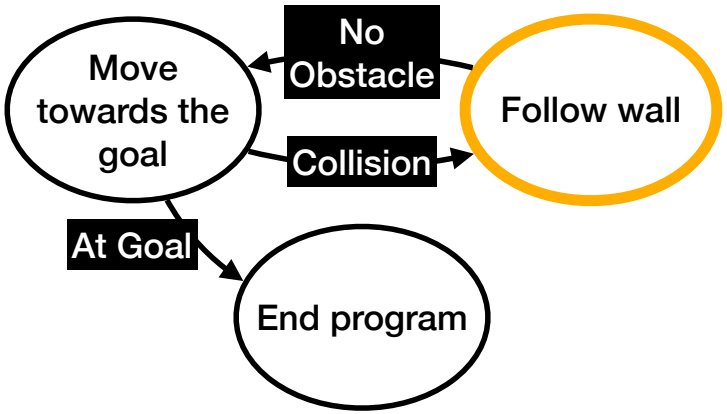
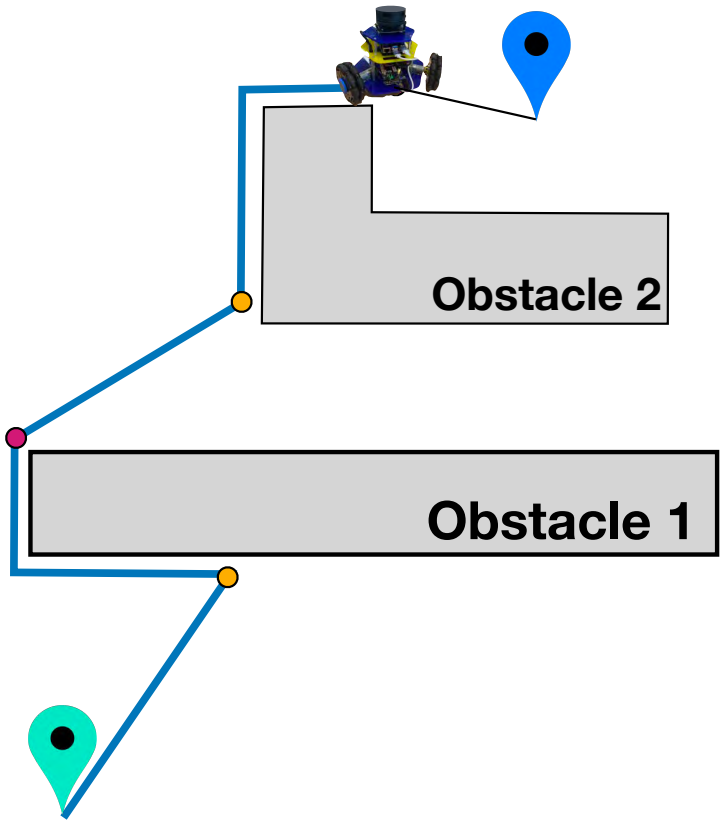




Follow wall until there is a line to the goal

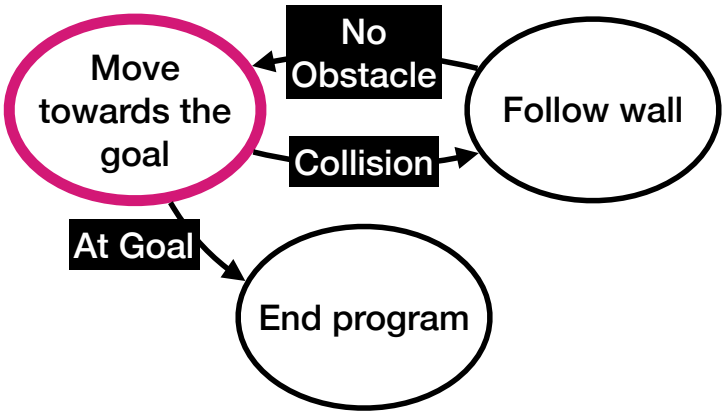
Bug algorithm

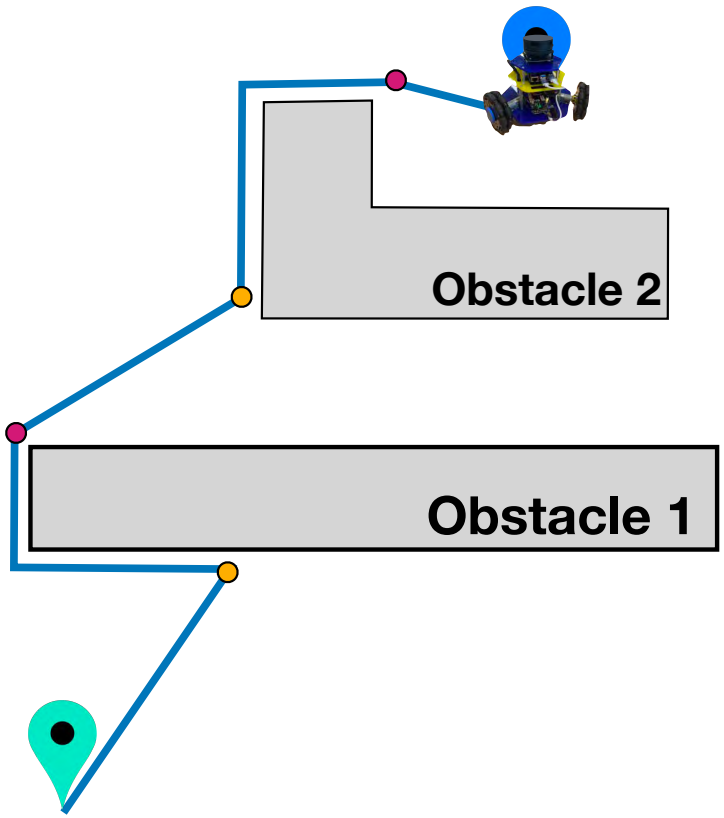




Once a line to goal is available, Move towards goal again

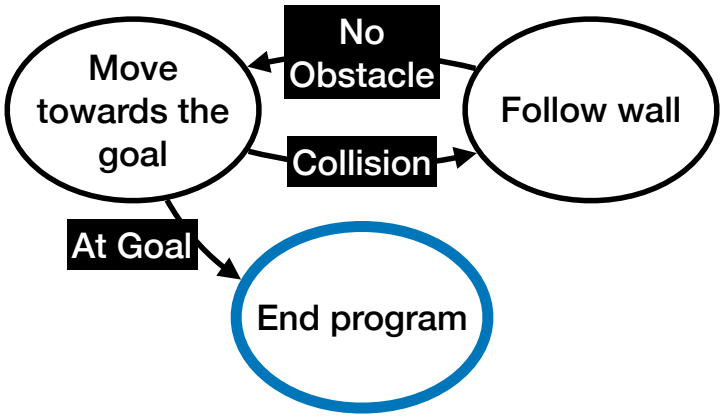
Bug algorithm

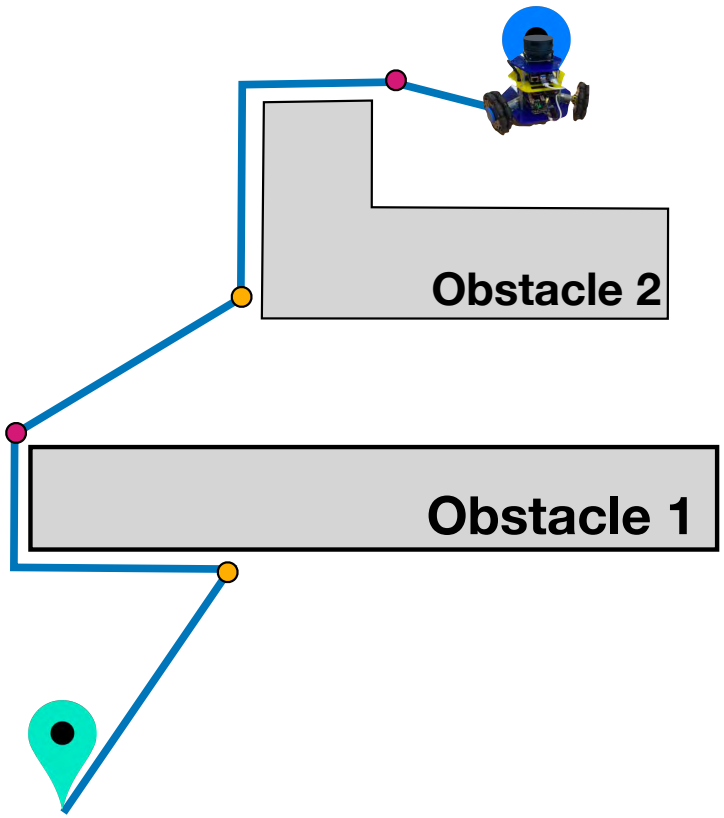




End when goal is reached

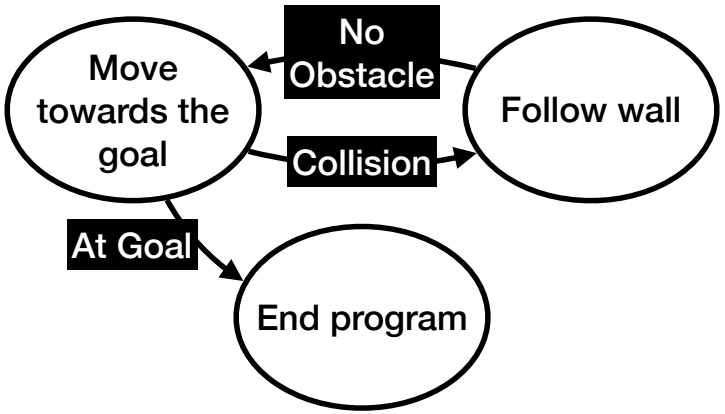
Bug algorithm

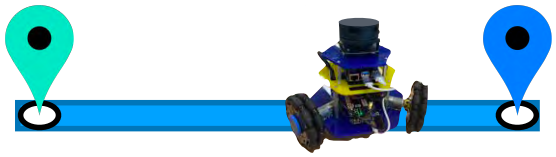




End when goal is reached

Bug algorithm

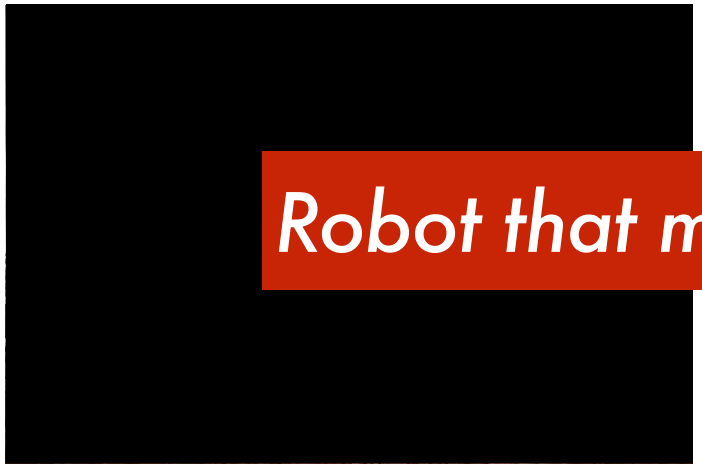


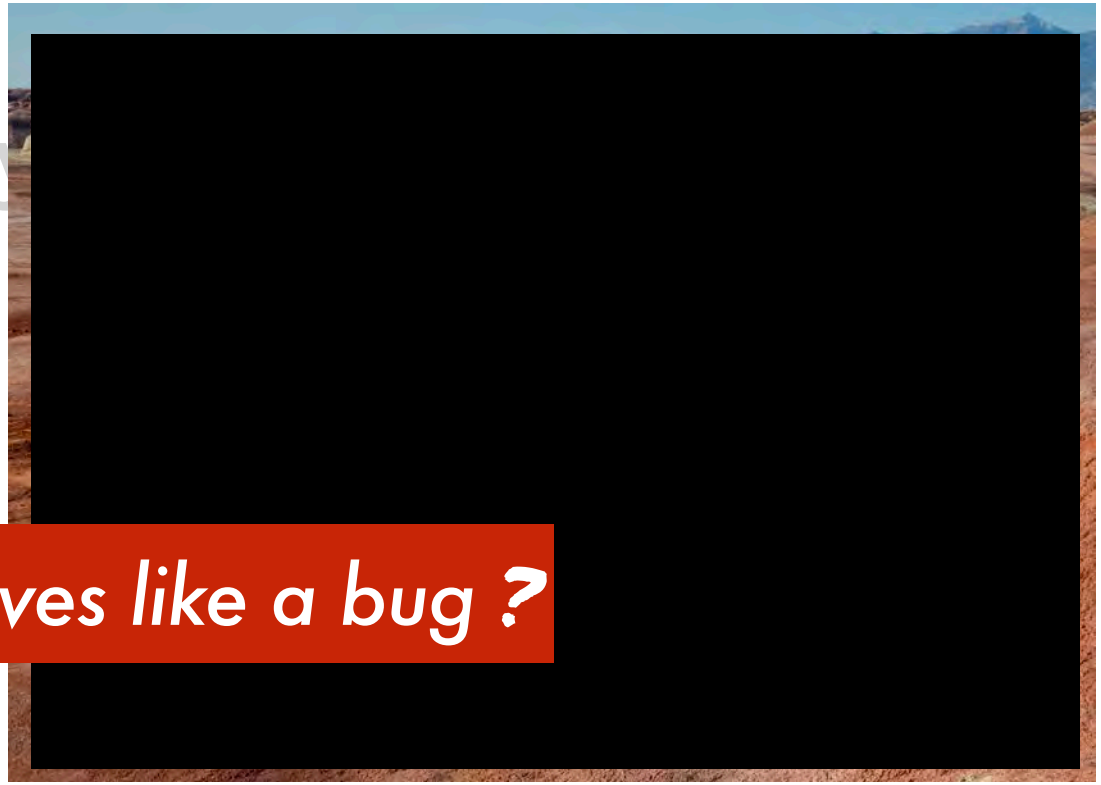


What options do we have for navigating our robot?

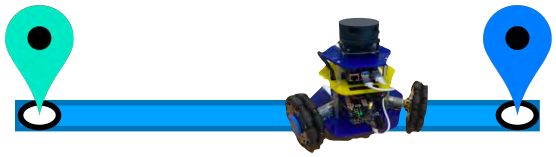
Just move randomly

Follow wall to goal





Robot that moves like a bug ?



What options do we have for navigating our robot?

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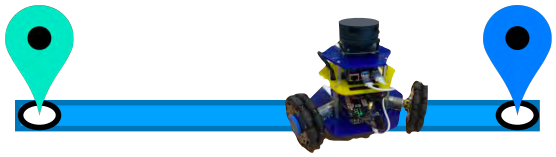
Follow wall to goal

Any benefits to bug motion?



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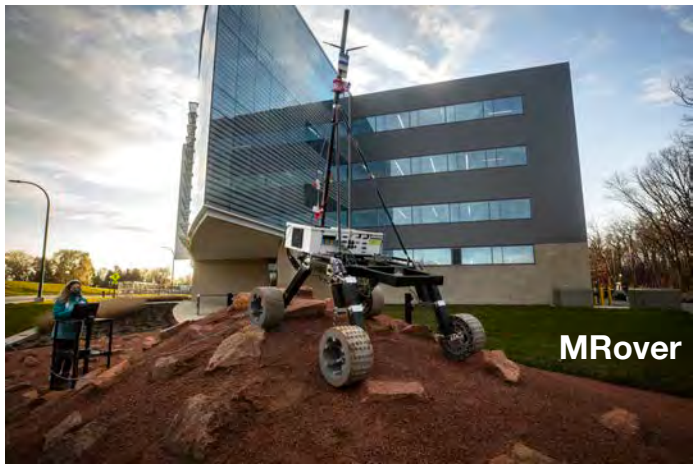
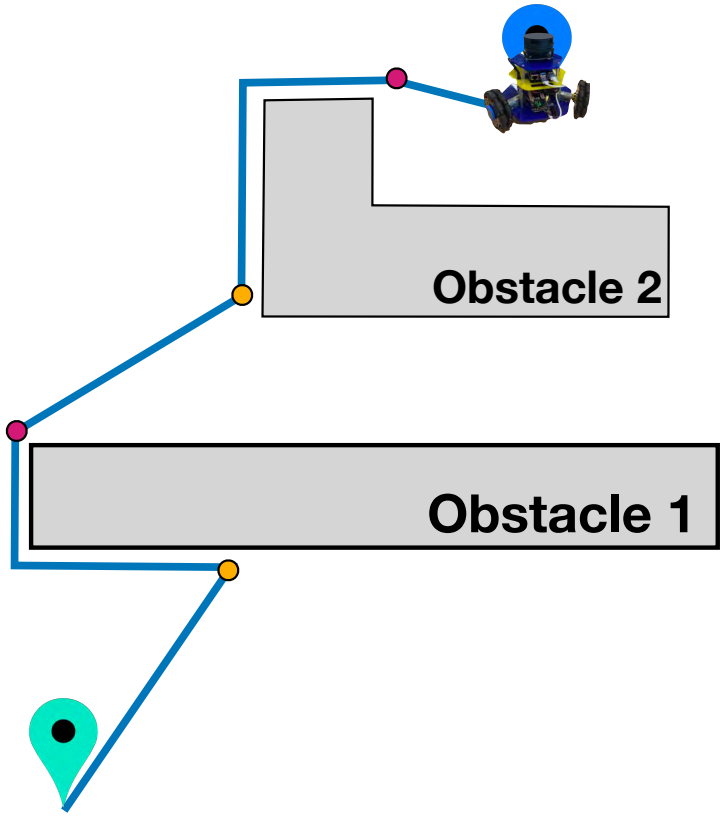


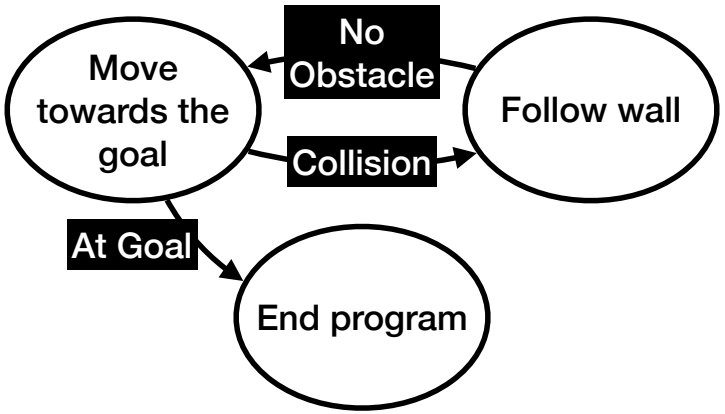
+ Simple
+ Reliable
  (reacts to its current situation)

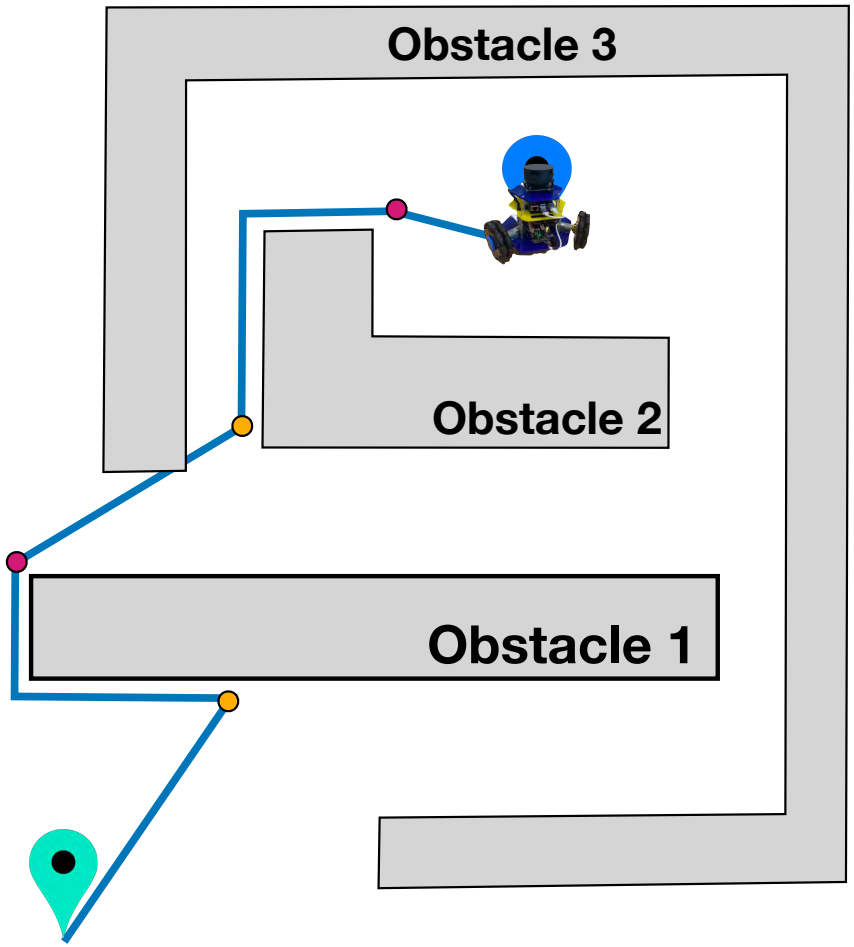
– Known goal location
  (assume we have GPS)
– Forgetful
  (reacts to its current situation)



Suppose we add an obstacle

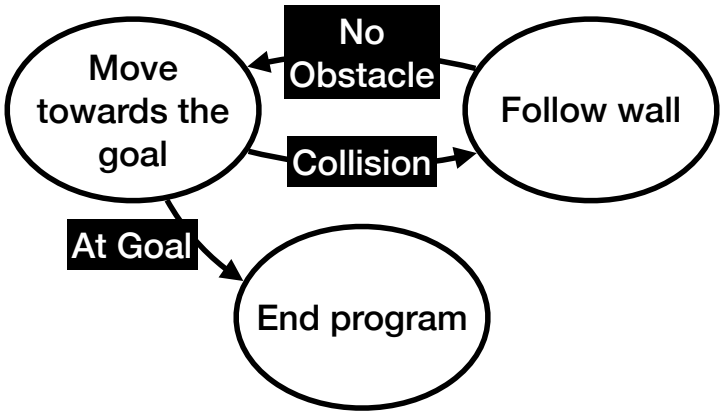
Bug algorithm

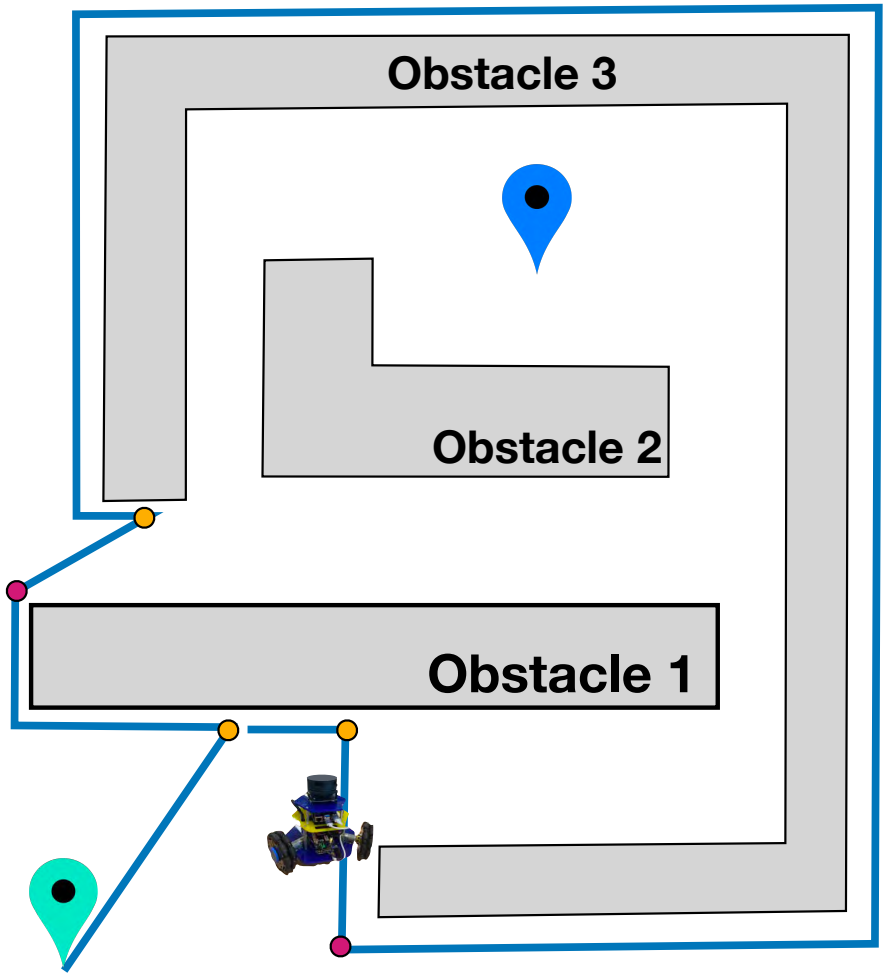




How would the result change ?

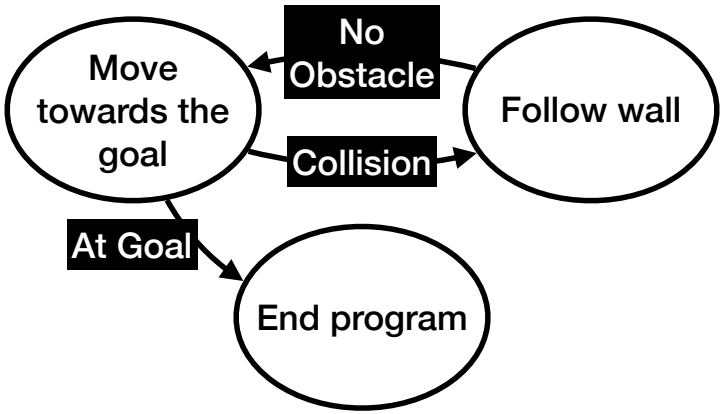
Bug algorithm

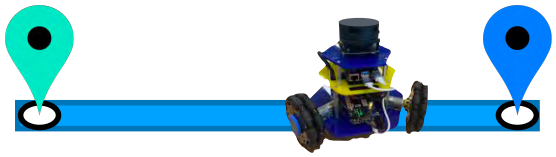




This program in this environment will never end

Bug algorithm



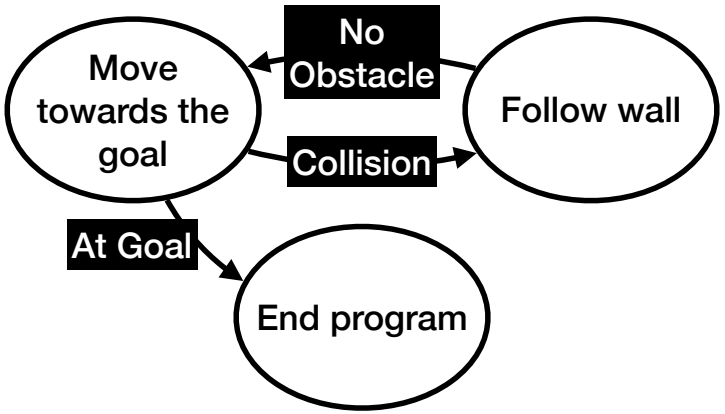


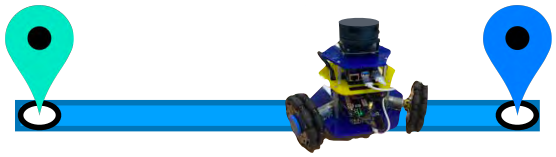
What options do we have for navigating our robot?

Just move randomly

Follow wall to goal

This program in this environment will never end





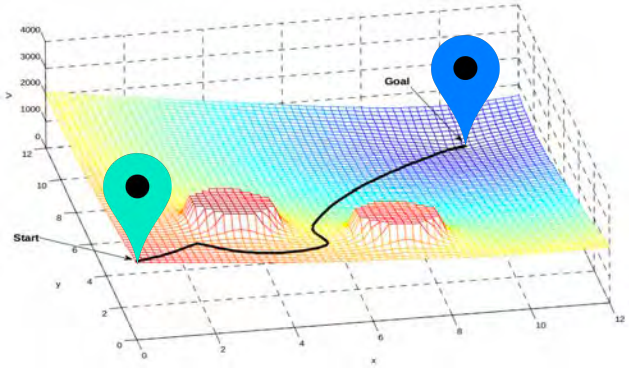
Just move randomly

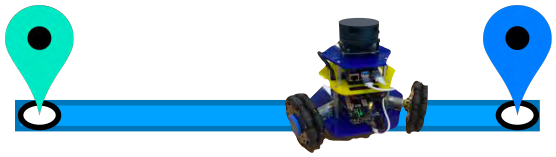
Follow wall to goal

Build a map to guide us

Project 2: Potential Fields

Autonomous navigation to a goal location





Just move randomly

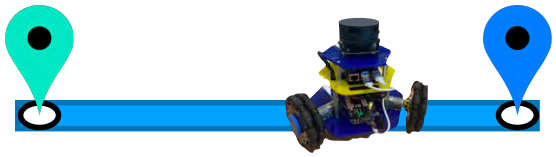
Follow wall to goal

Build a map to guide us

The map could express a hill: potential energy to the goal

Robot navigates like a marble: follows map potential to the goal

https://www.youtube.com/watch?v=UQGAlb_hss8



Just move randomly

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Project 2: Potential Fields

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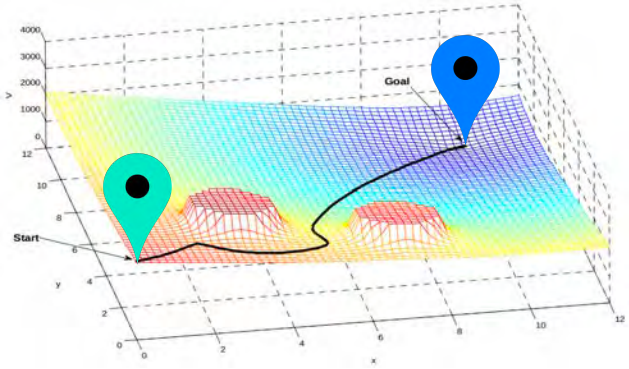
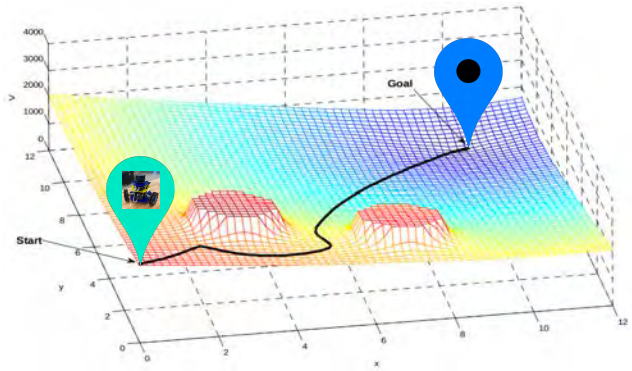


Project 2: Potential Fields

Autonomous navigation to a goal location

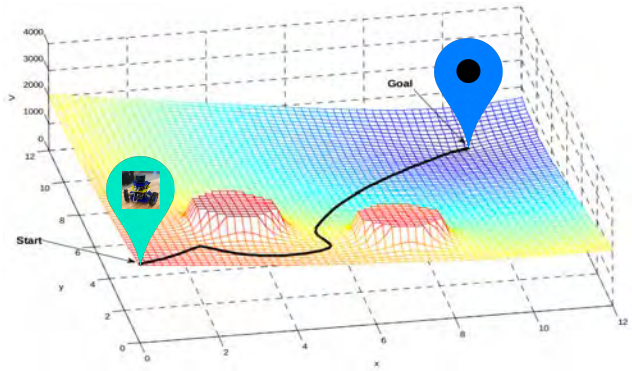


- [ ] Build map of environment
- [ ] Form attraction potential to goal
- [ ] Form repulsion potentials away from obstacles
- [ ] Add potentials together into potential field
- [x] Local search over potential field to navigate

Assume robot is like a marble. It will follow your course

Project 2: Potential Fields

Autonomous navigation to a goal location



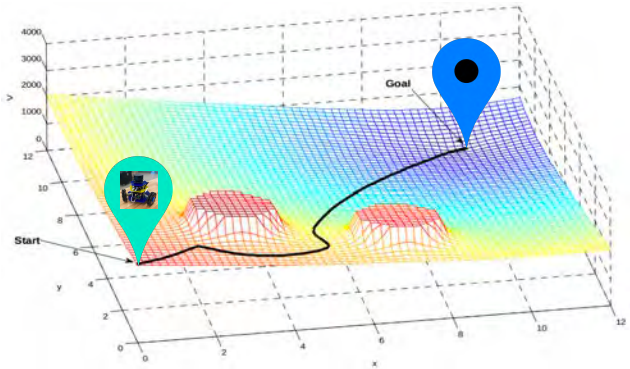
You have to build the course

- ☐ Build map of environment
- ☐ Form attraction potential to goal
- ☐ Form repulsion potentials away from obstacles
- ☐ Add potentials together into potential field
- ☑ Local search over potential field to navigate

Assume robot is like a marble.
It will follow your course

Project 2: Potential Fields

Autonomous navigation to a goal location

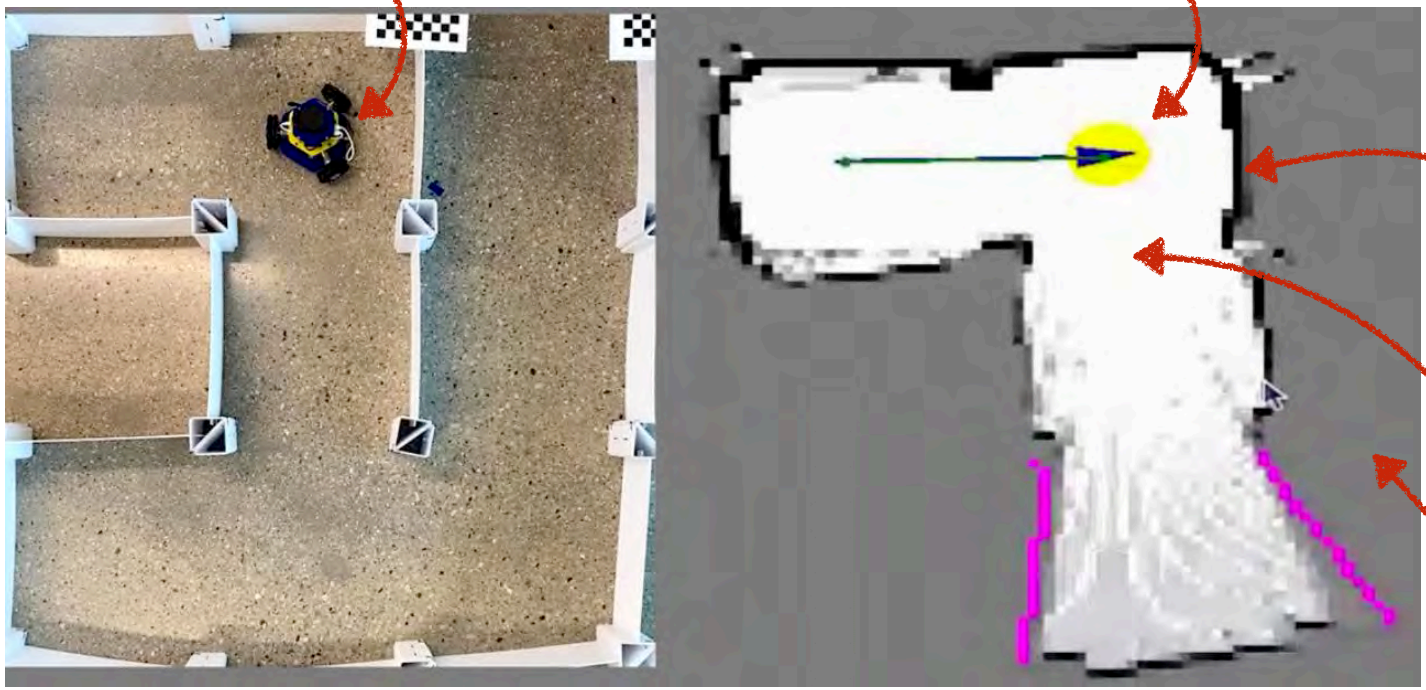


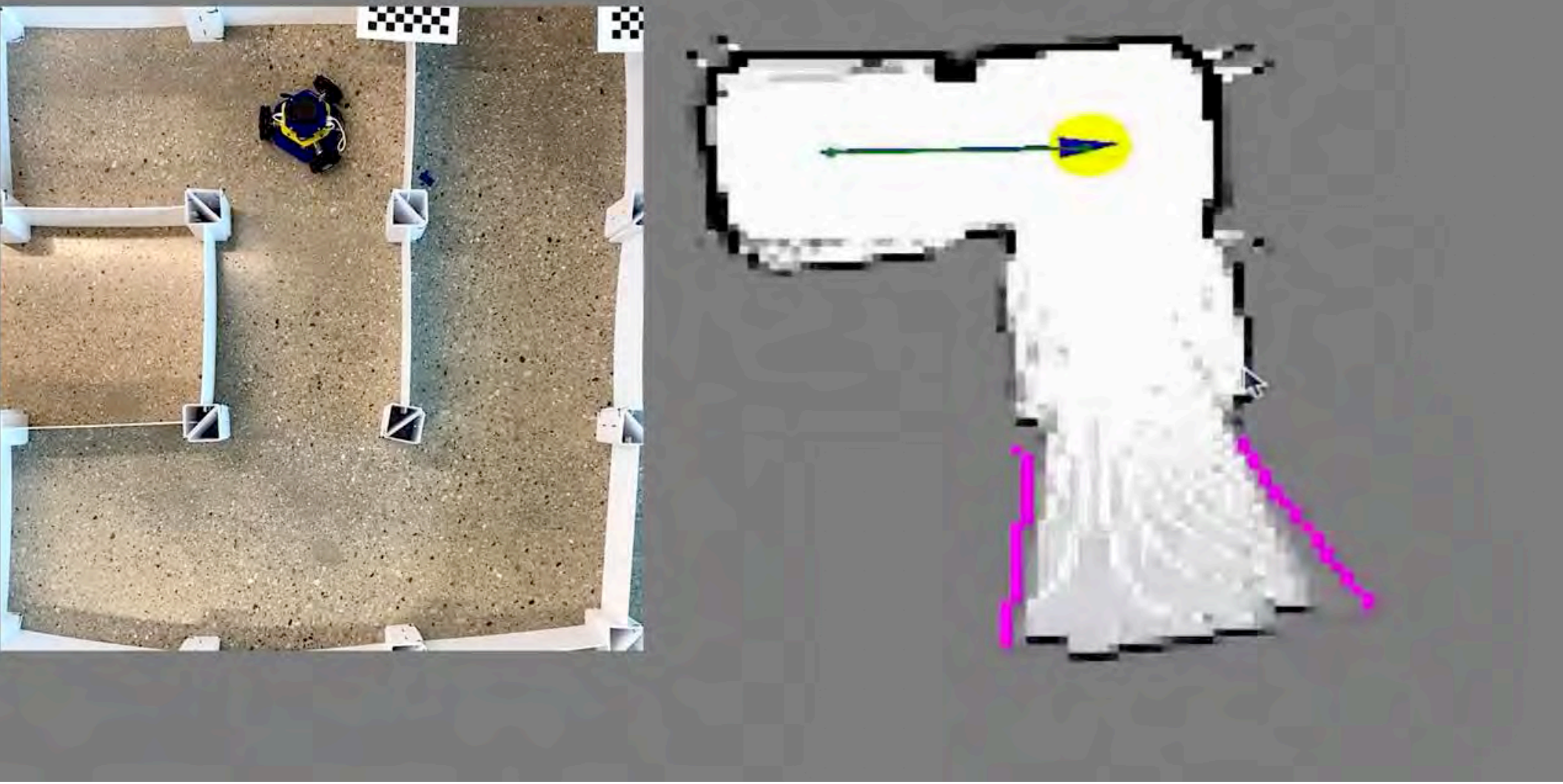
Use SLAM to build map

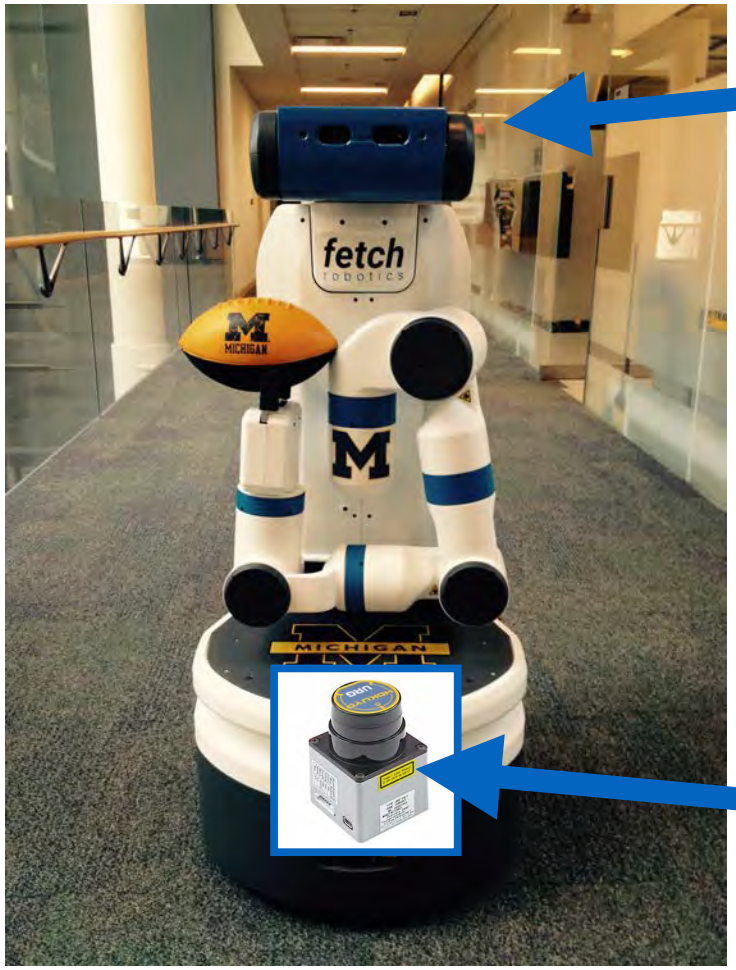
- [ ] Build map of environment
- [ ] Form attraction potential to goal
- [ ] Form repulsion potentials away from obstacles
- [ ] Add potentials together into potential field
- [x] Local search over potential field to navigate

Remember our first week of class

SLAM
Simultaneous Localization and Mapping



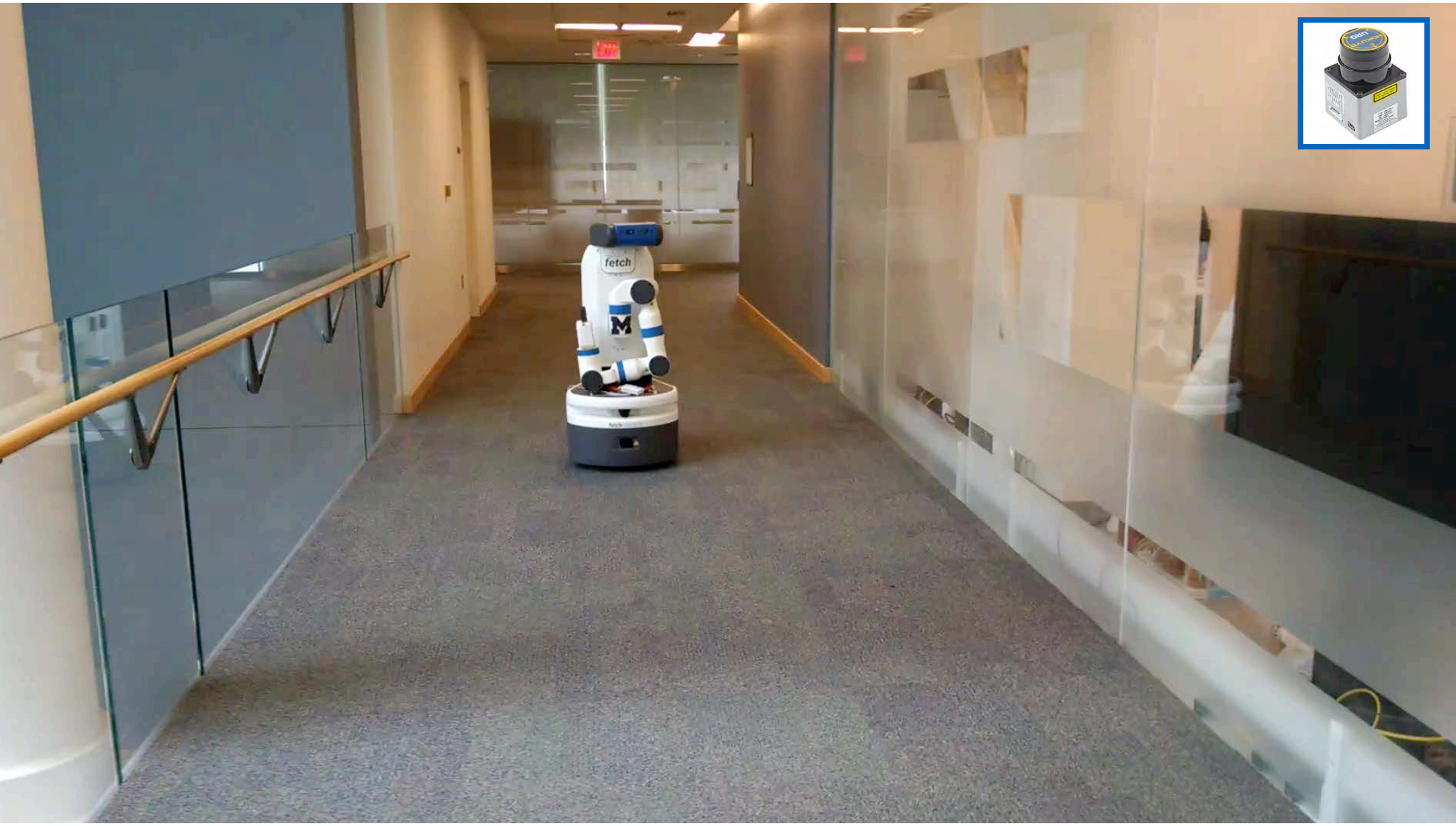




Color+Depth Camera

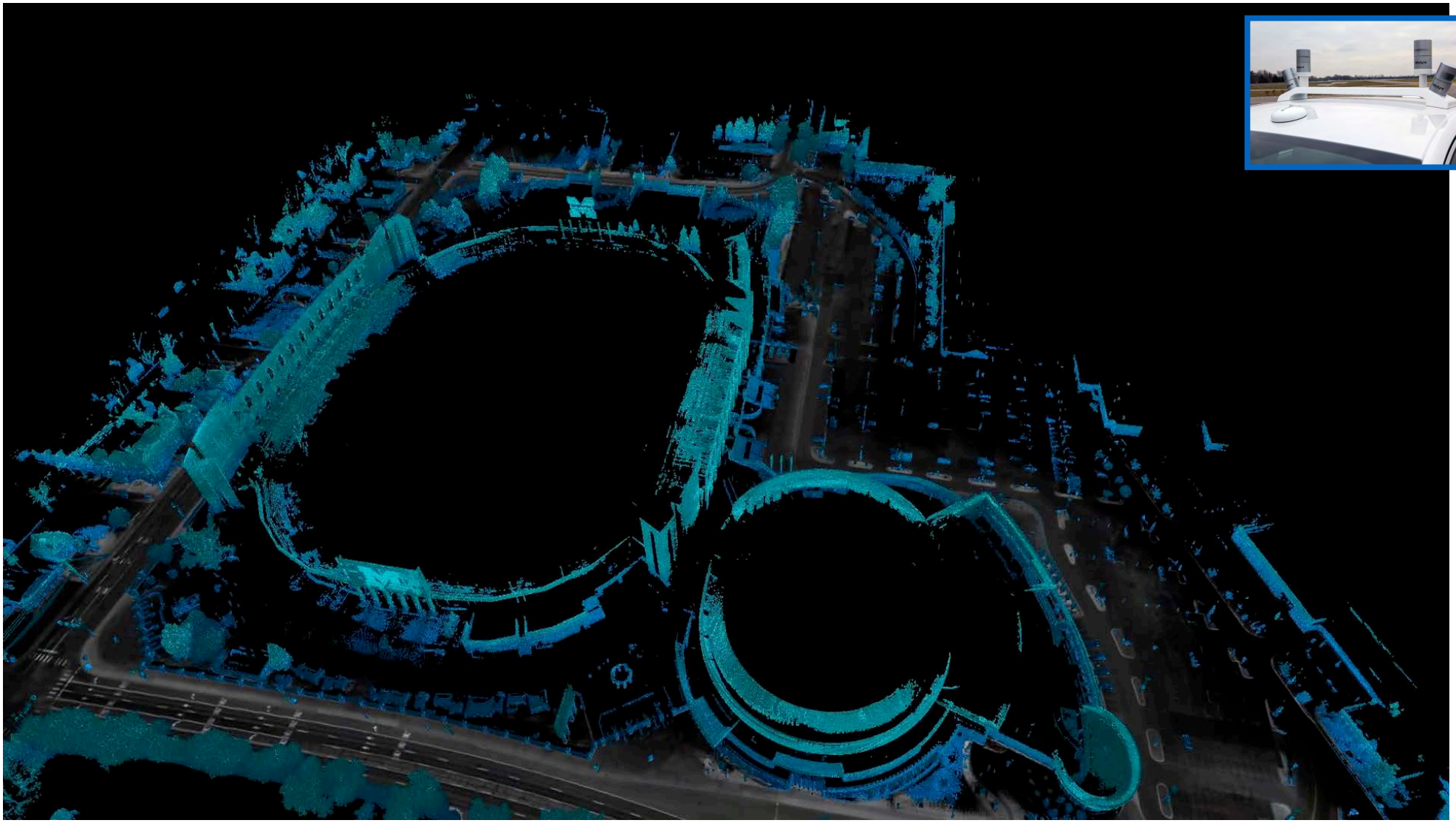
Laser Rangefinder


fetch
robotics
MICHIGAN
M
MICHIGAN
fetchrobotics


fetch

EECS 467 - Winter 2021 - Enclosure Escape Assignment - Ko et al.
https://www.youtube.com/watch?v=mZtdOlbTbvU&list=PLDutmfAv2lfZc2DQVNHfNODWsokz85OJA&index=14

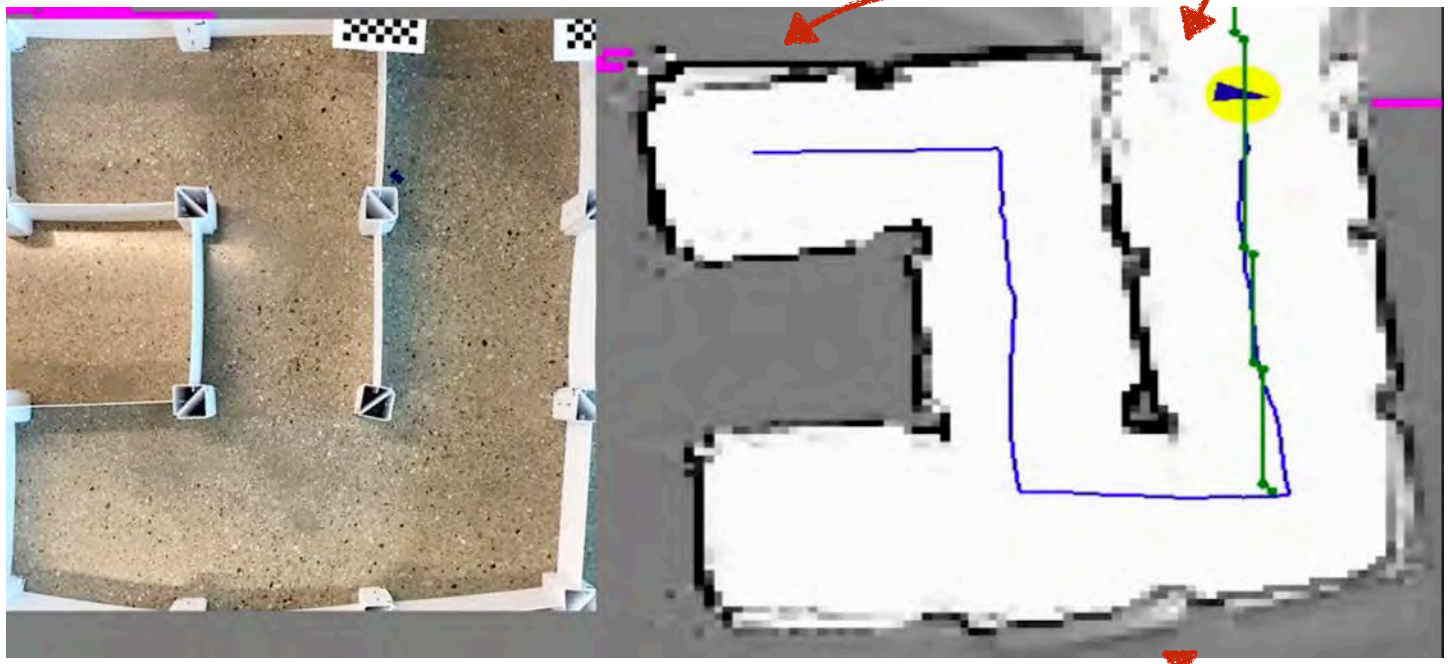
Michigan Next Generation Vehicle (Olson, Eustice, et al.)



Michigan Next Generation Vehicle (Olson et al.)

Be careful !!!

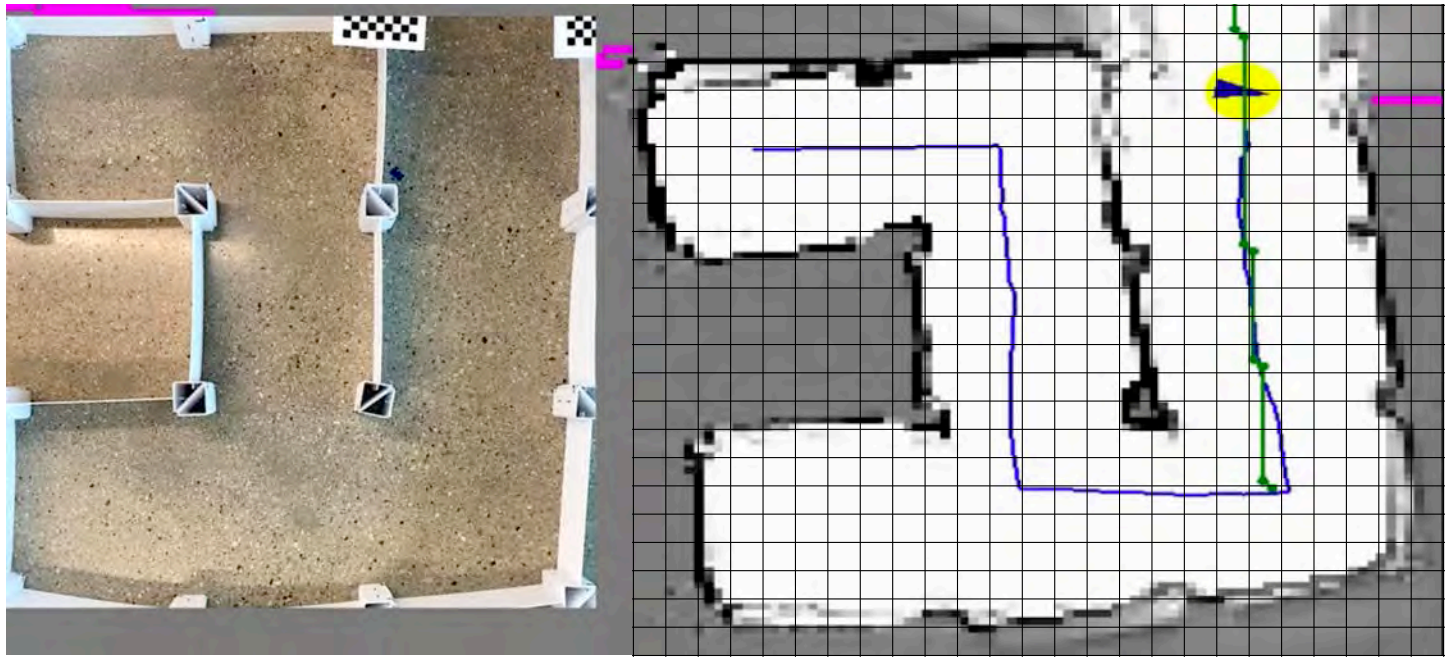
KiMBot SLAM is susceptible to noise



Real world

SLAM output

Robot map is stored as an image and represented as a graph



Real world ***SLAM output***

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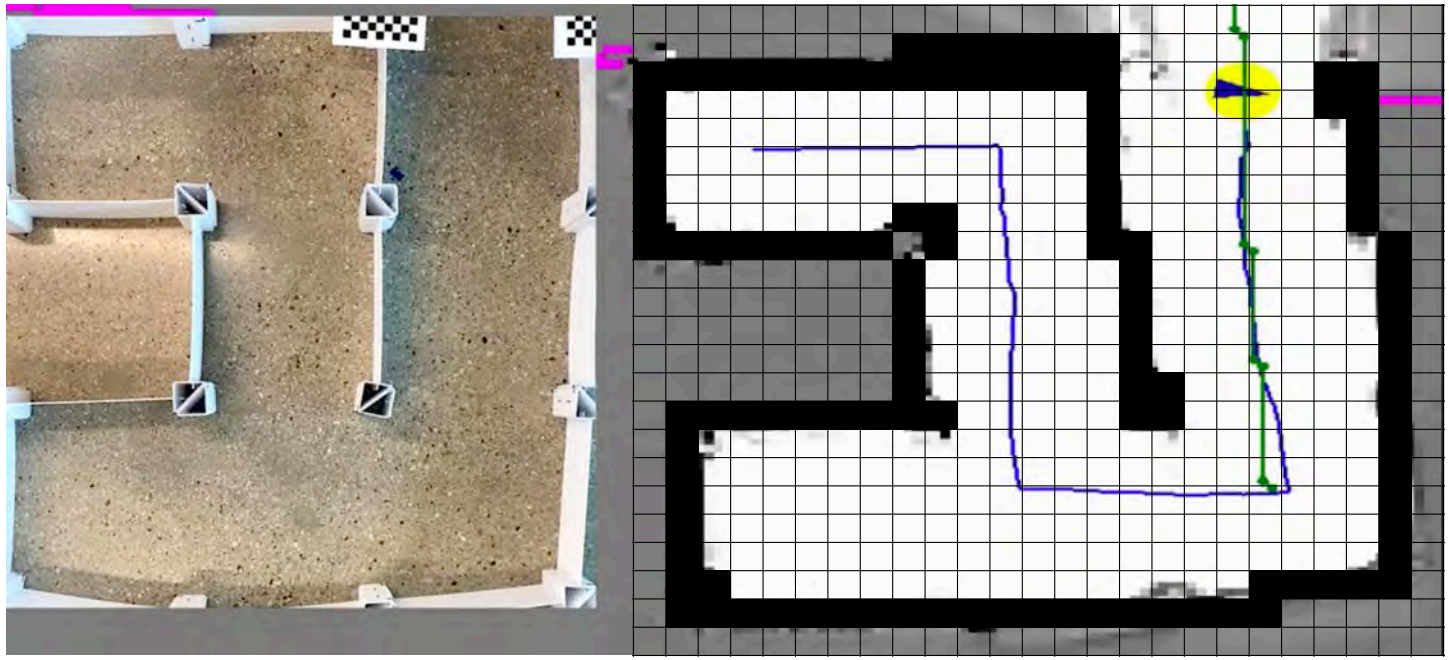

Real world *SLAM output*

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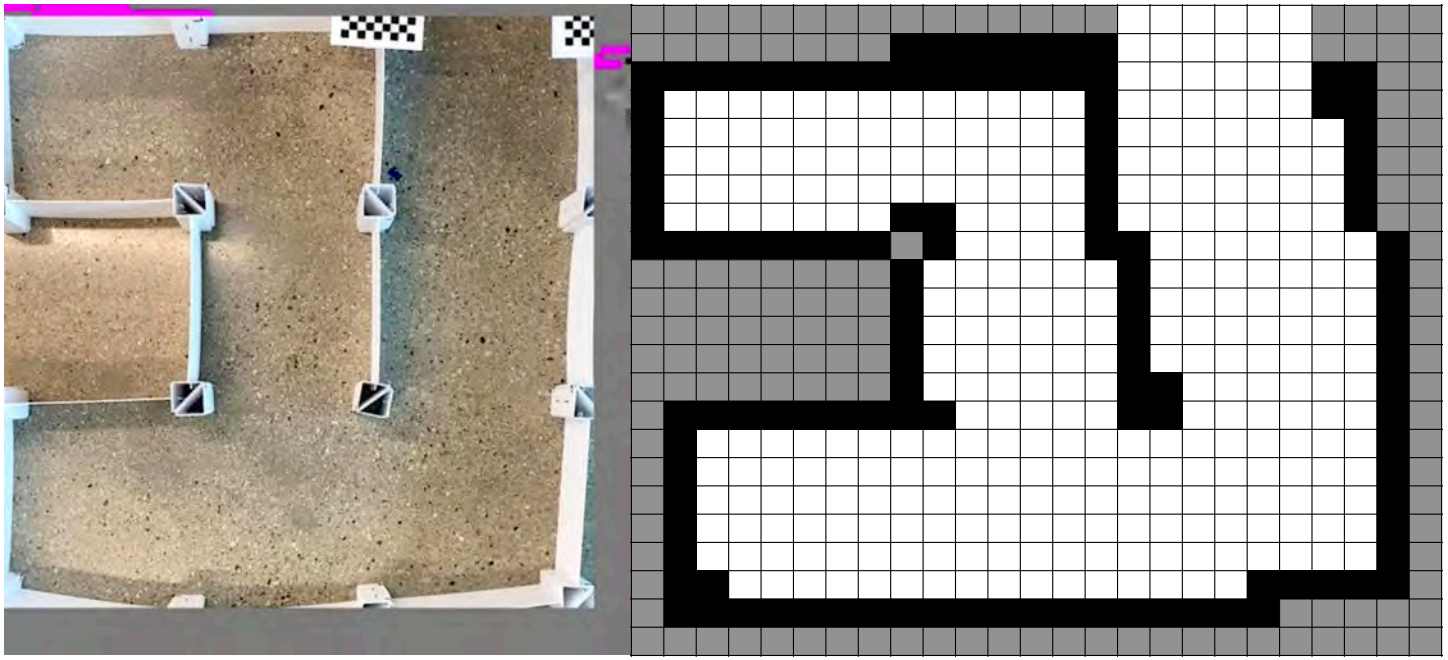
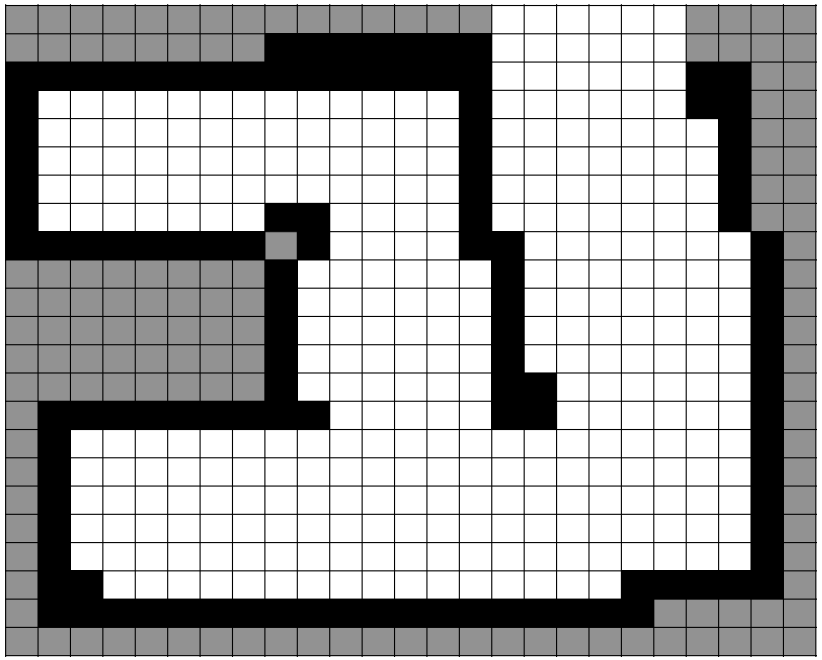


Real world *SLAM output*

Robot map is stored as an image and represented as a graph

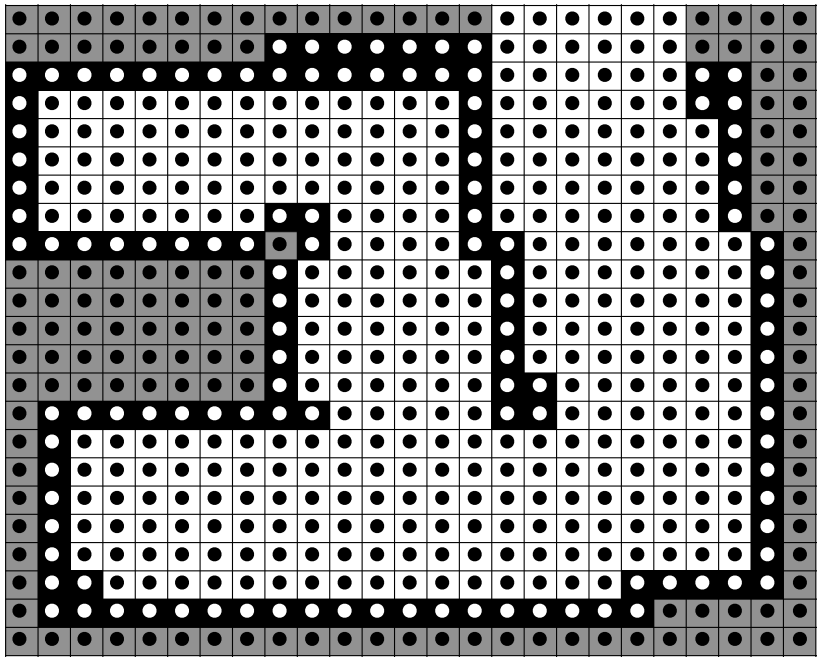
A vector of cells over robot locations



Robot map is stored as an image and represented as a graph

A vector of cells over robot locations

Every cell has a node in the graph



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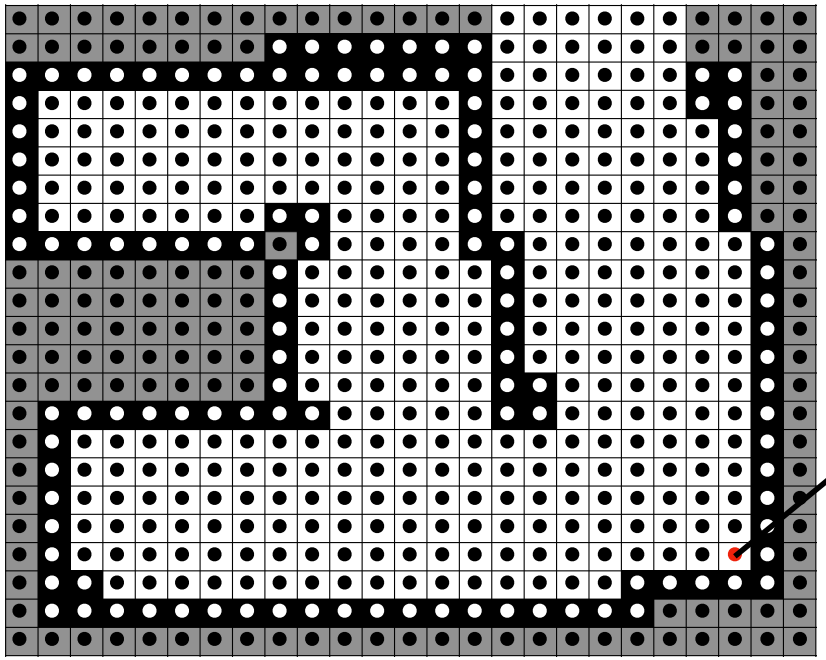


```
origin_x: 2.2
origin_y: 0.3
occupied: true
```

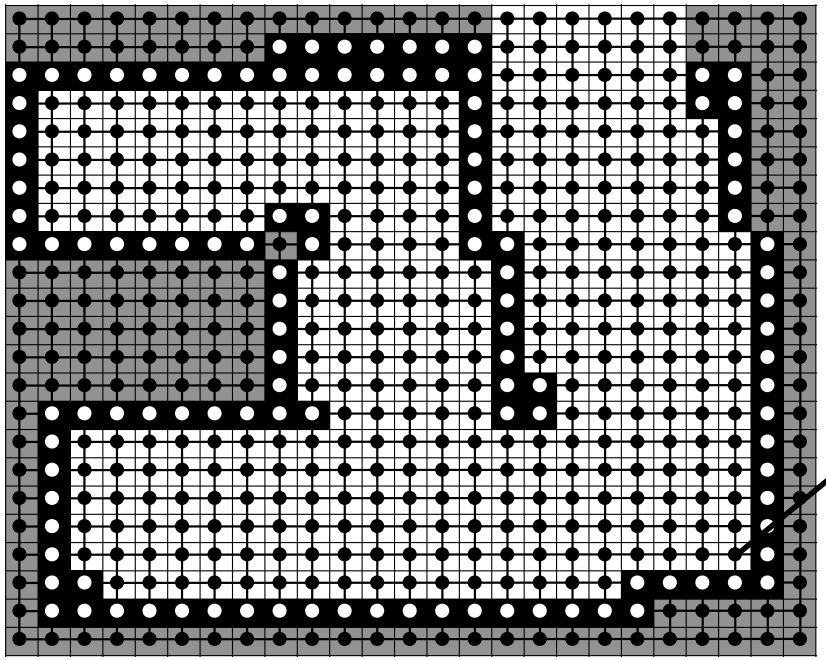
A graph node stores a struct of information about the cell

Robot map is stored as an image and represented as a graph

A vector of cells over robot locations

Every cell has a node in the graph

Every pair of neighboring cells shares an edge in the graph



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origin_x: 2.2
origin_y: 0.3
occupied: true
```

A graph node stores a struct of information about the cell

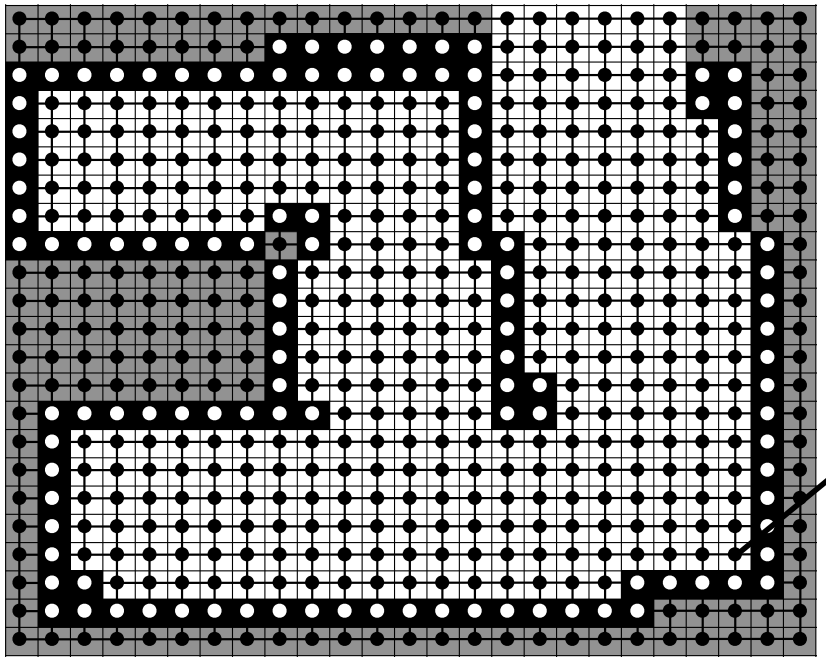
- [ ] Form attraction potential to goal
- [ ] Form repulsion potentials away from obstacles
- [ ] Add potentials together

Potentials can express the navigation cost of a node

A vector of cells over robot locations

Every cell has a node in the graph

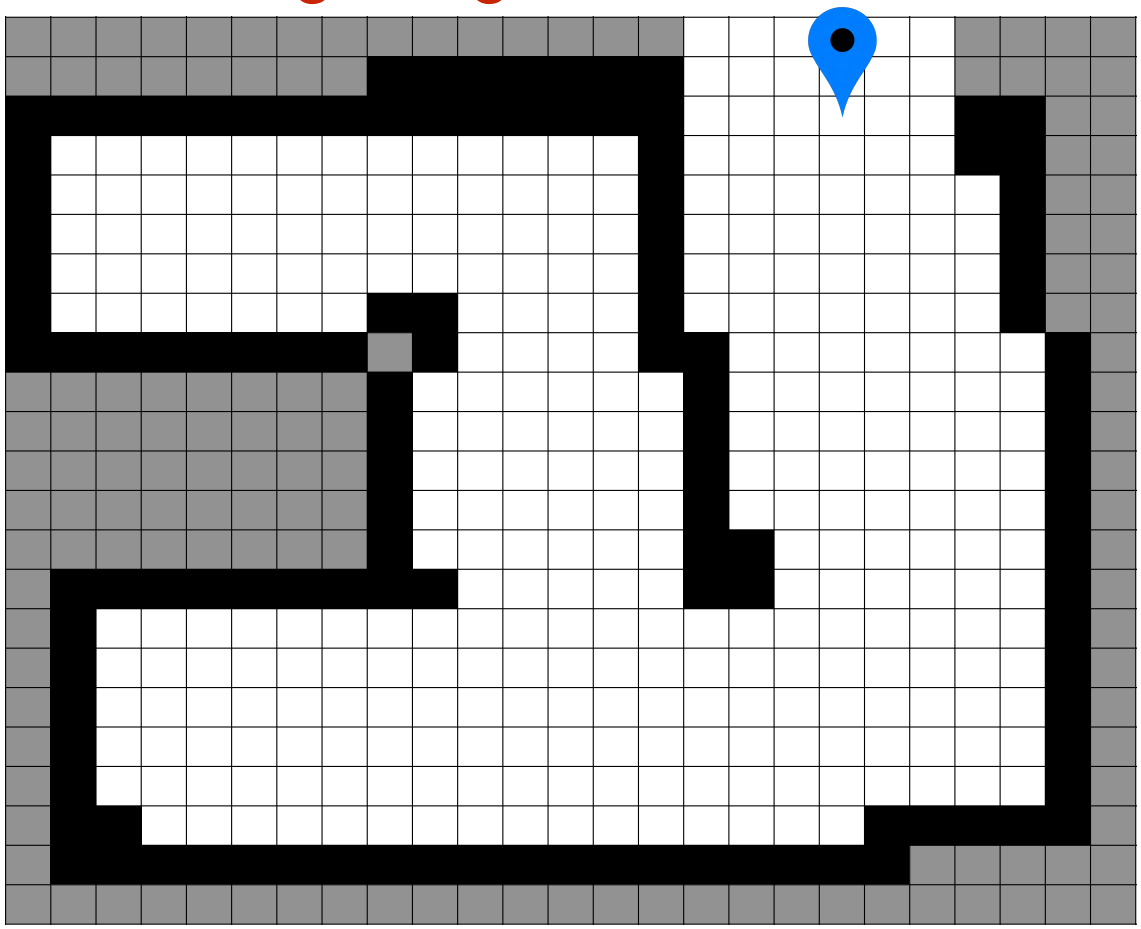
Every pair of neighboring cells shares an edge in the graph



```
origin_x: 2.2
origin_y: 0.3
occupied: true
cost: ??
```

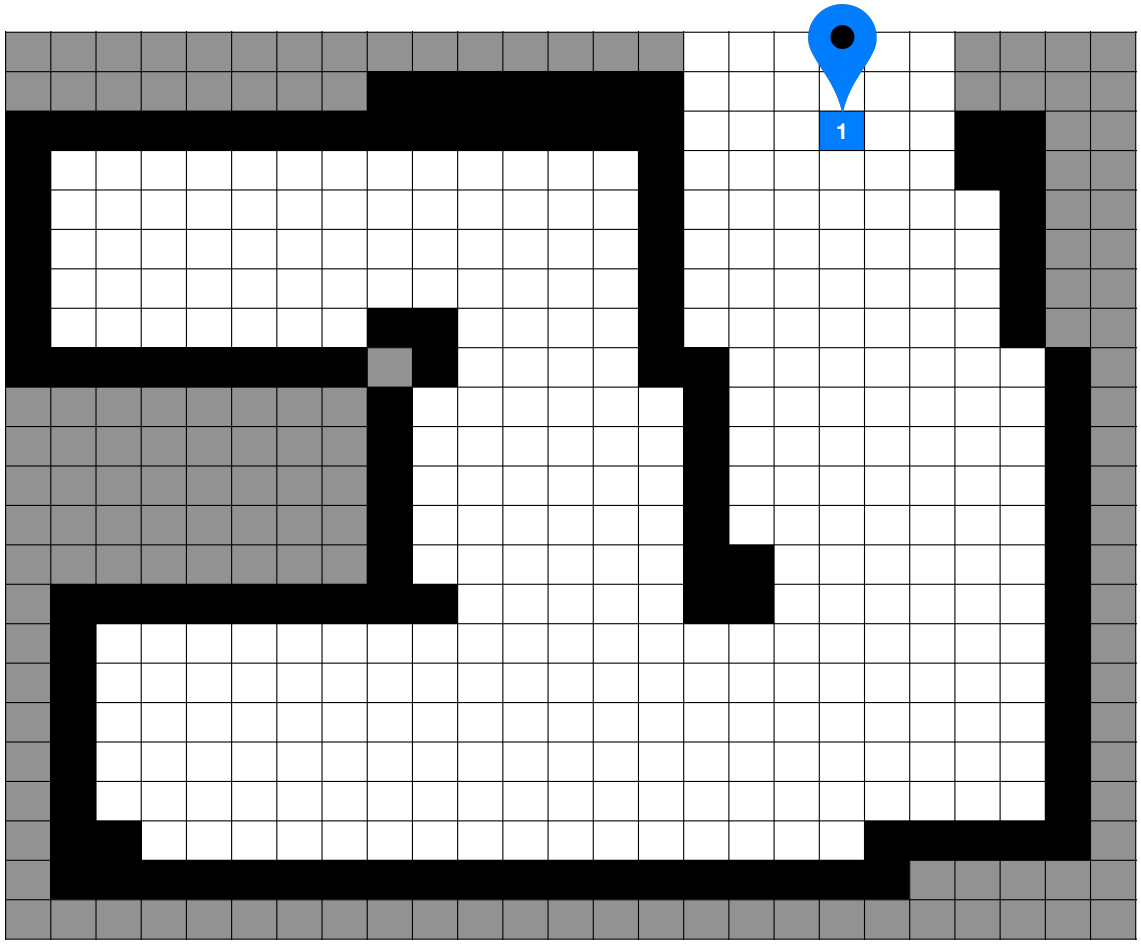
A graph node stores a struct of information about the cell

Assume robot is navigating to a given goal location



What could `.cost` ***look like at each node ?***

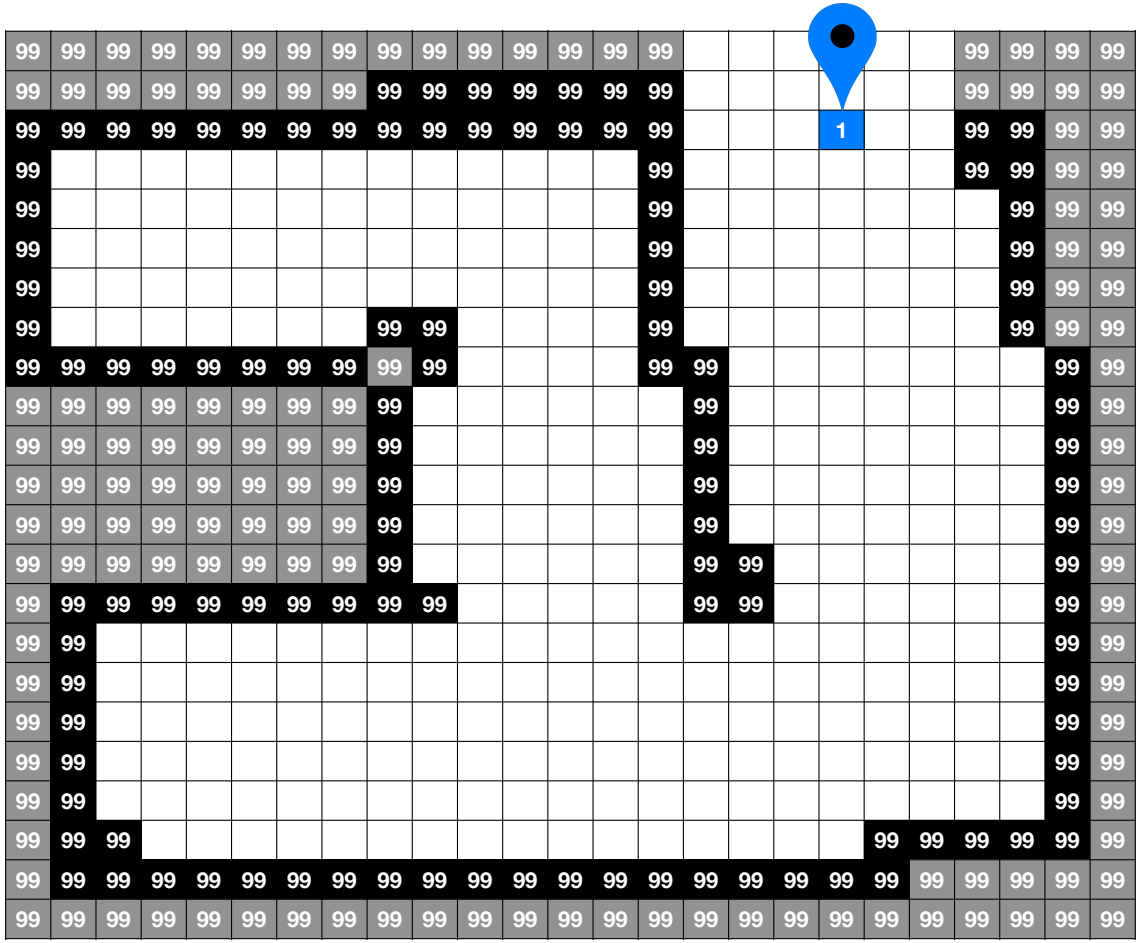
Low cost at goal



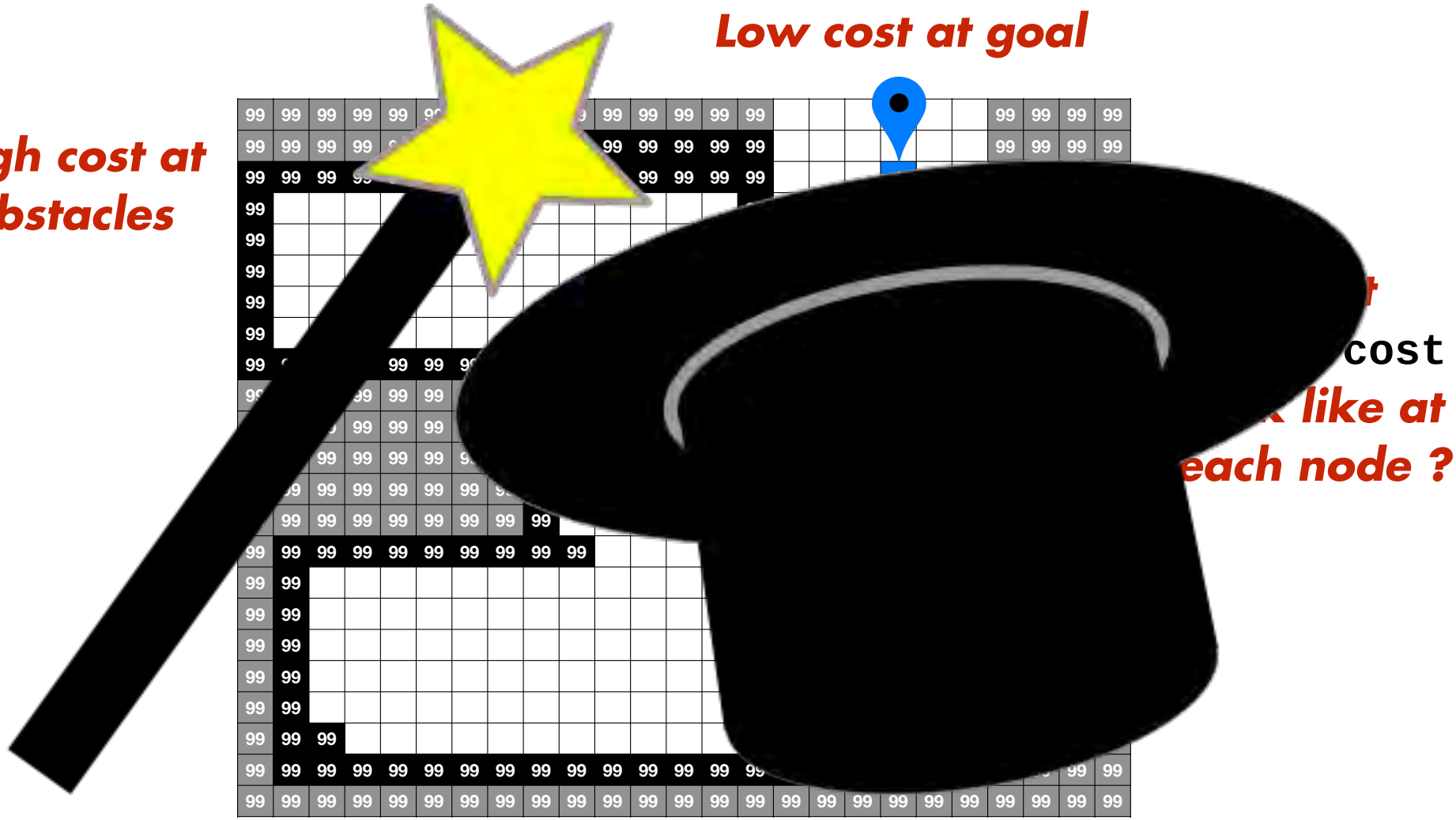
What could `.cost` ***look like at each node ?***

Low cost at goal

High cost at obstacles



What could `.cost` *look like at each node ?*


Low cost at goal
High cost at obstacles
cost
like at
each node ?

99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	8	6	4	6	90	99	99	99	99
99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	6	4	2	4	90	99	99	99	99
99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	4	2	1	2	90	99	99	99	99
99	90	90	90	90	90	90	90	90	90	90	90	90	90	99	90	6	4	2	4	90	99	99	99	99
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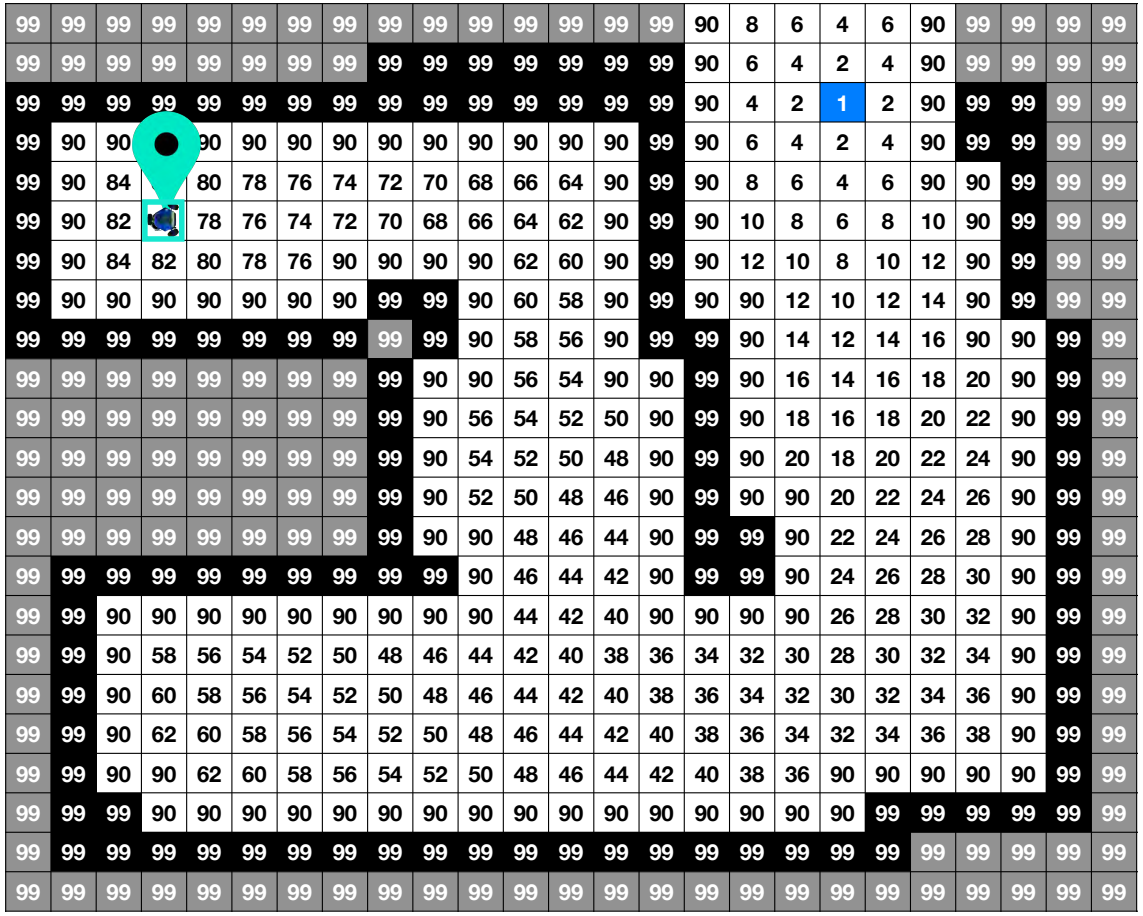
99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99

What could `.cost` look like at each node ?

Assume robot is navigating from a given start location



Assume robot is navigating from a given start location

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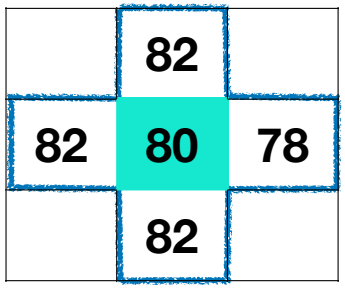
Search locally for next best move



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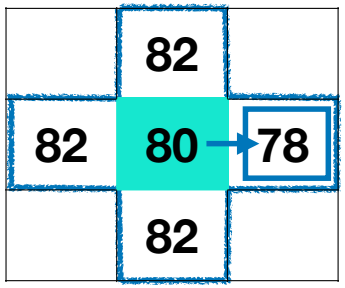
Search locally for next best move



Neighbor node with lowest potential

99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	8	6	4	6	90	99	99	99	99
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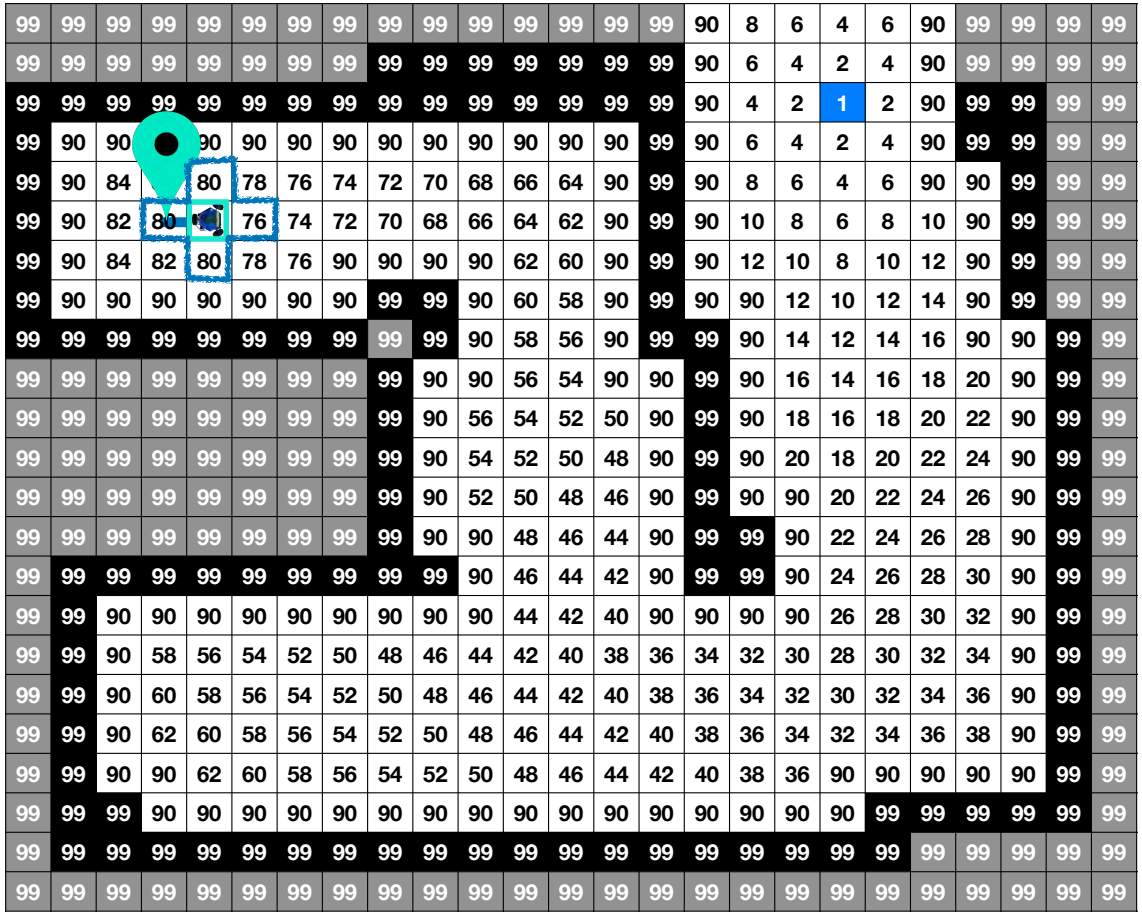
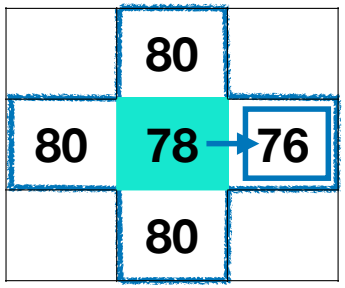
Search locally for next best move



Search locally for next best move



Neighbor node with lowest cost

99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	8	6	4	6	90	99	99	99	99
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99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	4	2	1	2	90	99	99	99	99
99	90	90	90	90	90	90	90	90	90	90	90	90	90	99	90	6	4	2	4	90	99	99	99	99
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99	90	82	80	78	76	74	72	70	68	66	64	62	90	99	90	10	8	6	8	10	90	99	99	99
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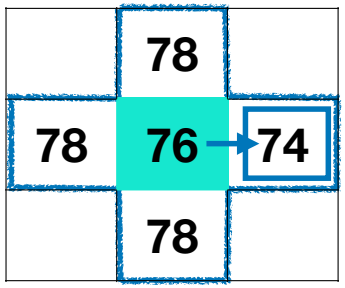
Search locally for next best move



Neighbor node with lowest cost

99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	8	6	4	6	90	99	99	99	99
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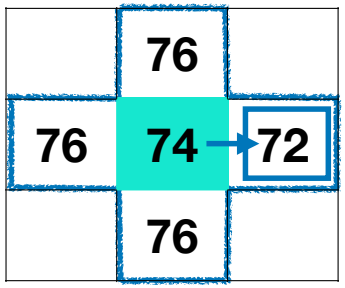
Search locally for next best move



Neighbor node with lowest cost

99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	8	6	4	6	90	99	99	99	99
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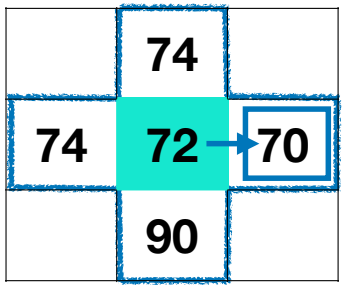
Search locally for next best move



Neighbor node with lowest cost

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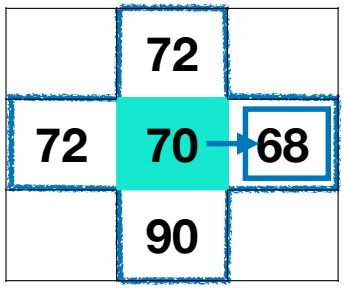
Search locally for next best move



Neighbor node with lowest cost

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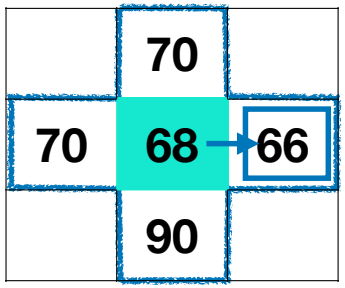
Search locally for next best move



Neighbor node with lowest cost

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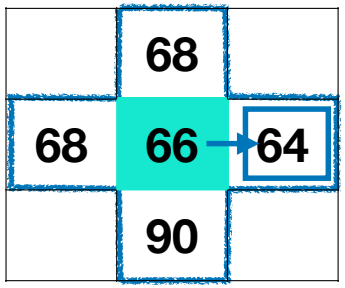
Search locally for next best move



Neighbor node with lowest cost

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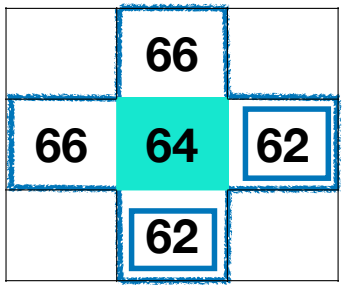
Search locally for next best move



Neighbor node with lowest cost

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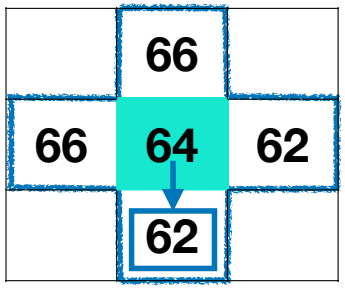
Search locally for next best move



A neighbor node with lowest cost

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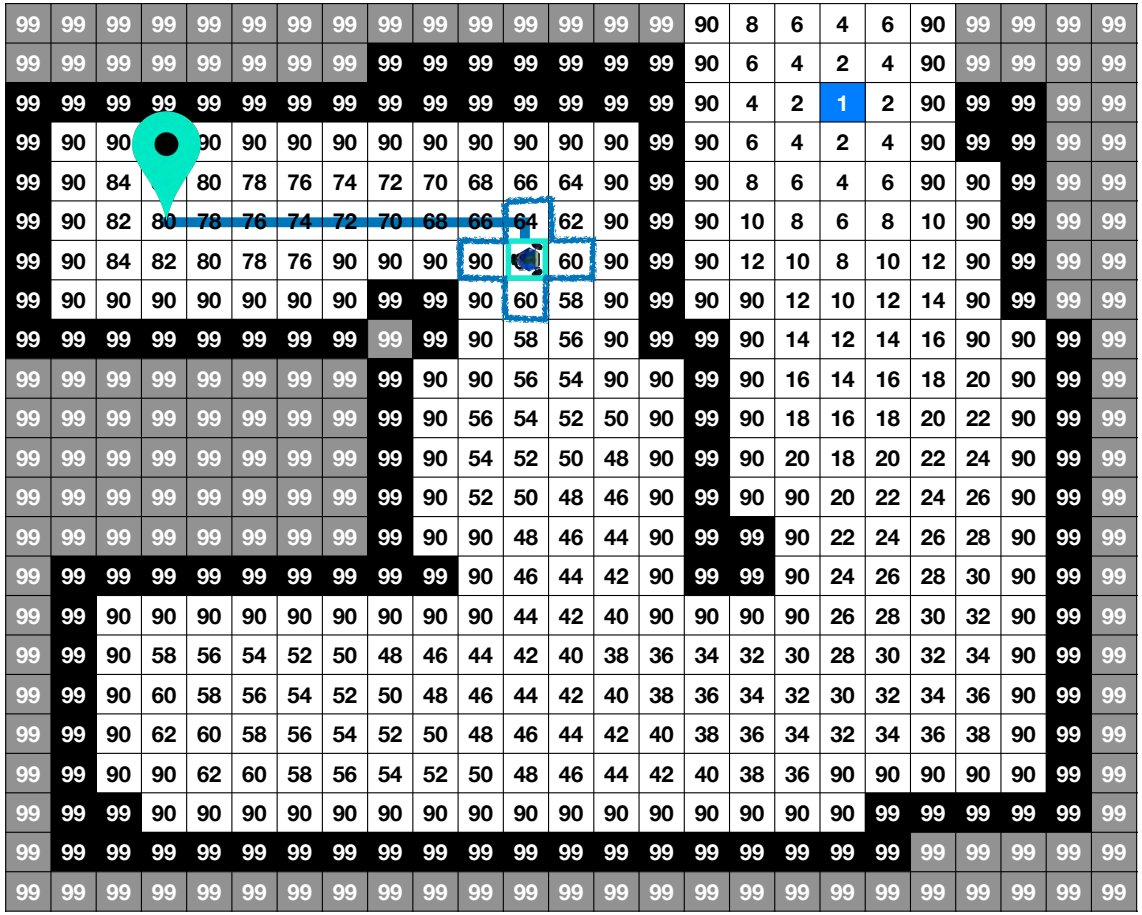
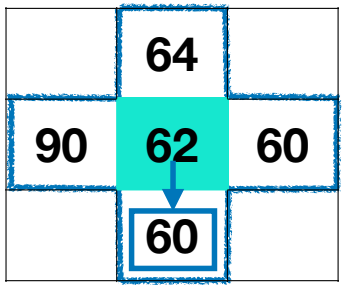
Search locally for next best move



Search locally for next best move



A neighbor node with lowest cost

99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	8	6	4	6	90	99	99	99	99
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99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	4	2	1	2	90	99	99	99	99
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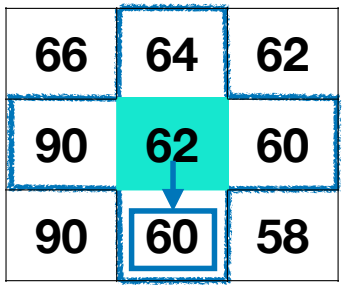
Search locally for next best move



A neighbor node with lowest cost

99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	8	6	4	6	90	99	99	99	99
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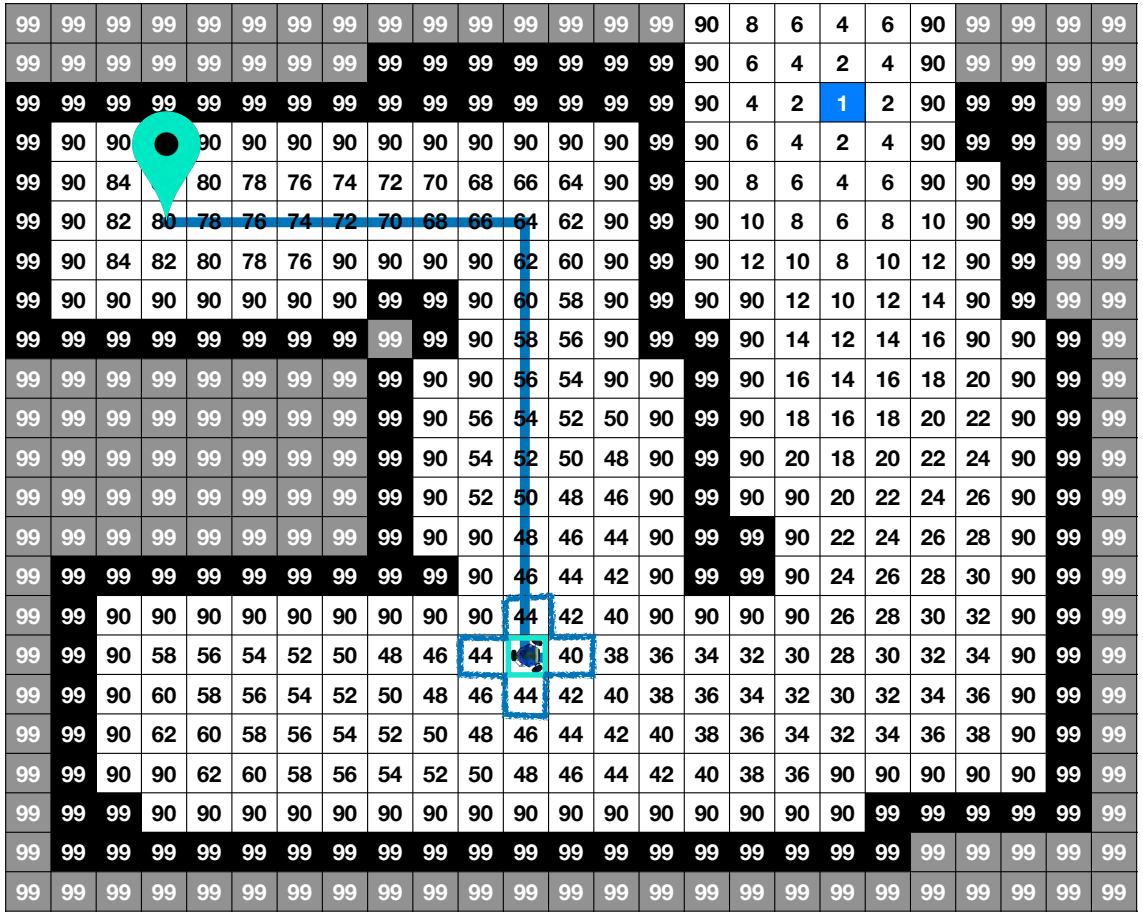
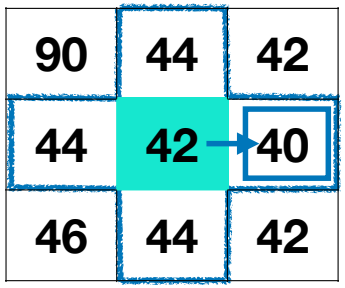
Search locally for next best move

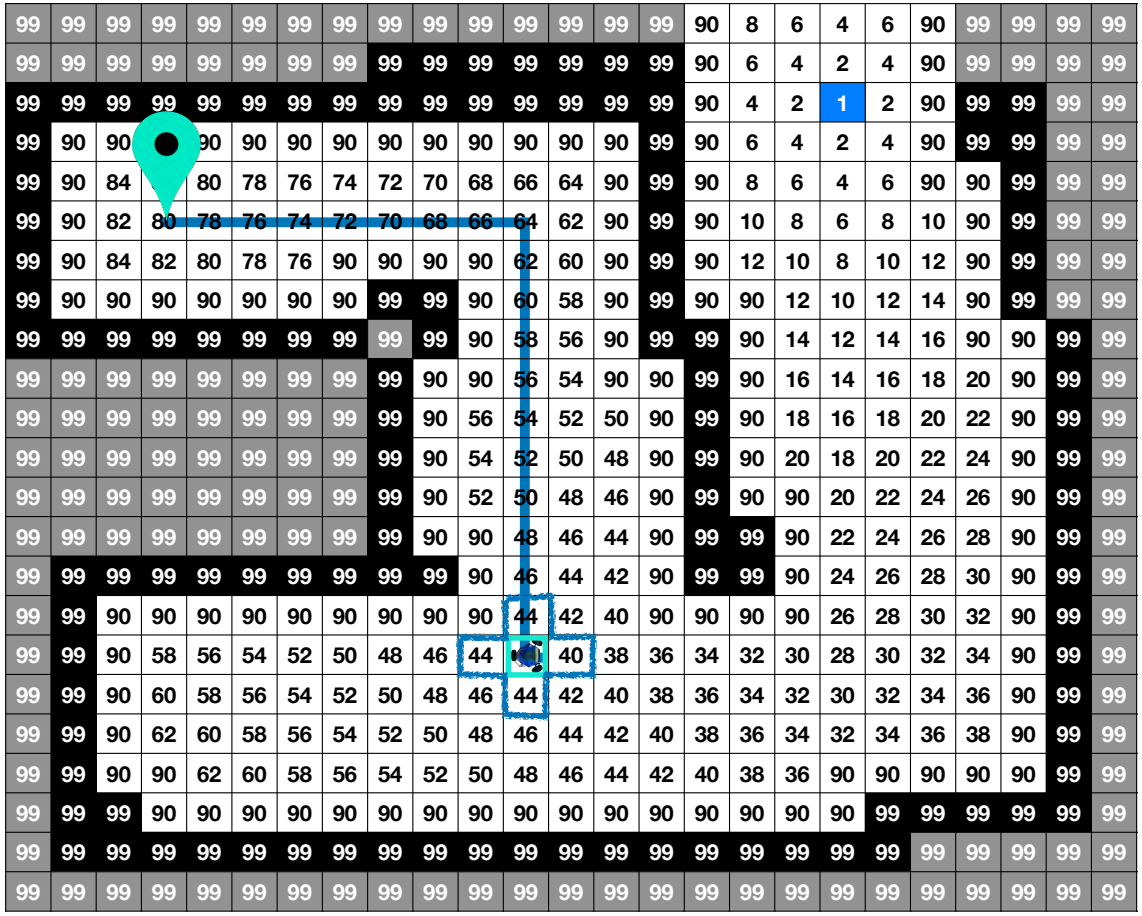
Forwarding ahead...



Search locally for next best move

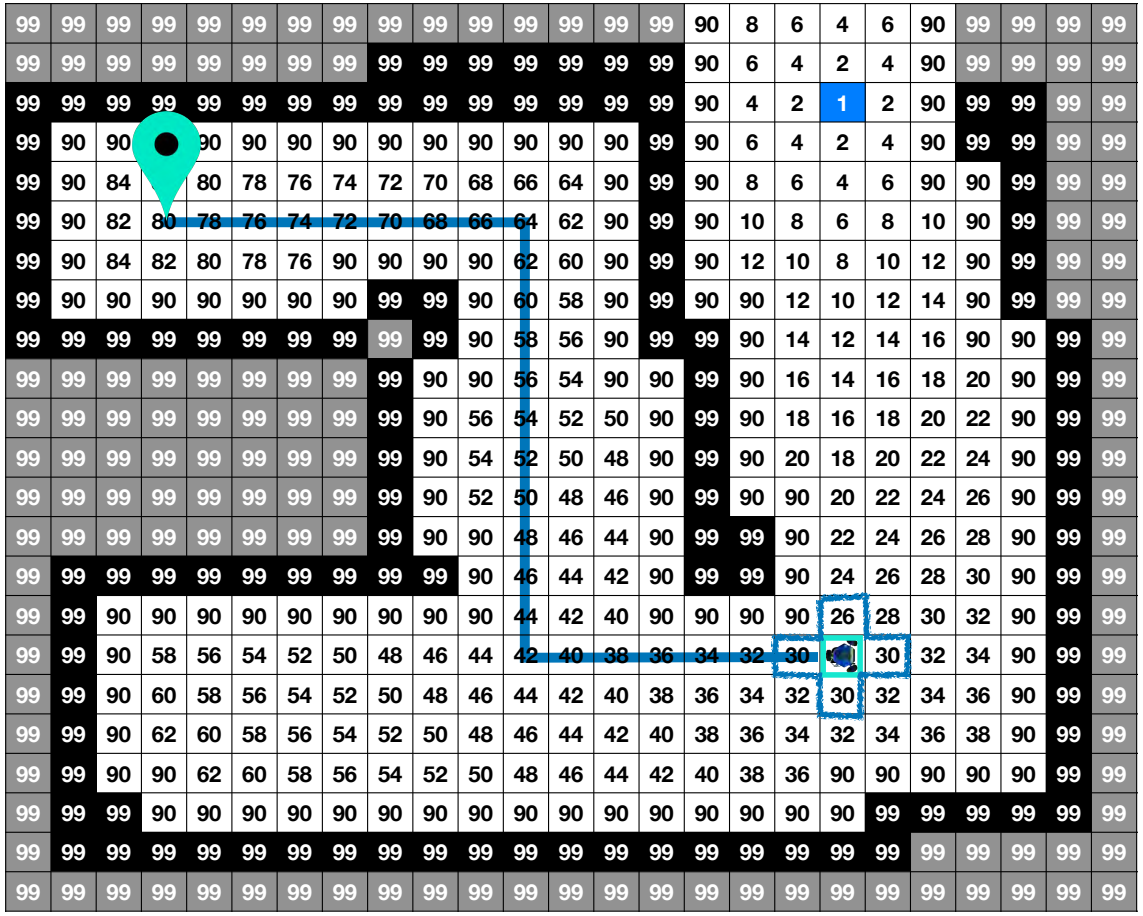


A neighbor node with lowest cost



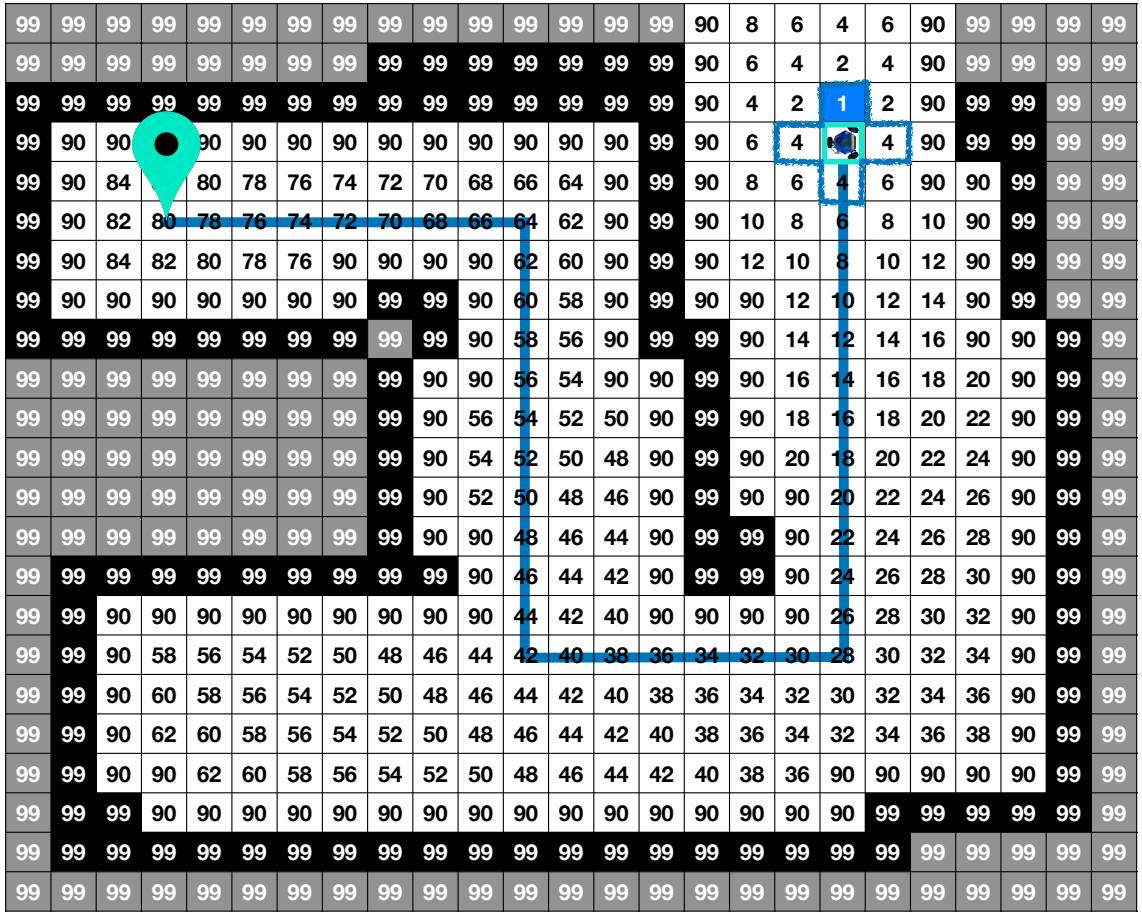
Search locally for next best move

Forwarding ahead...

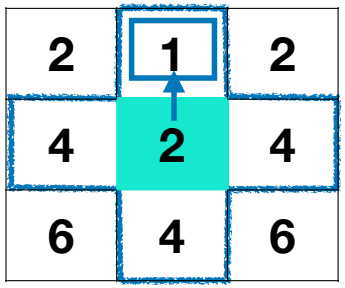


Search locally for next best move

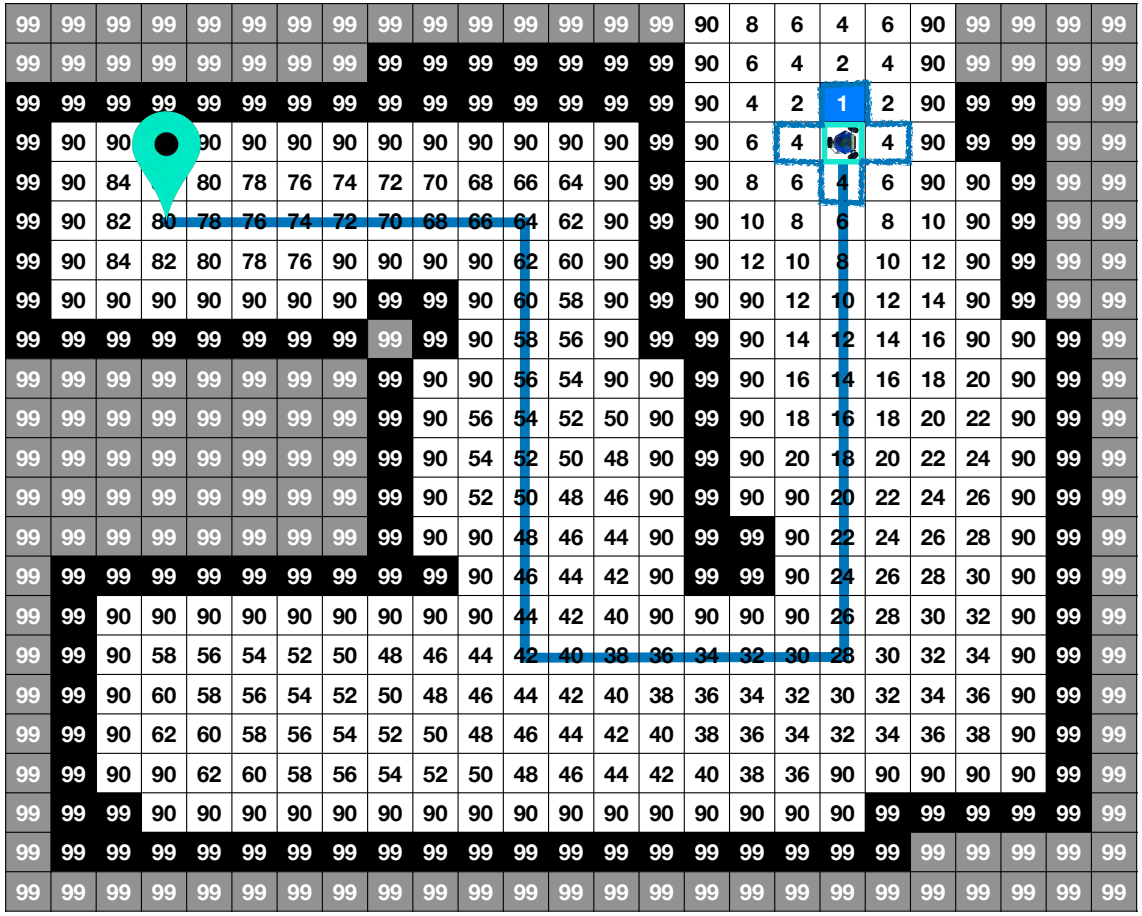
Forwarding ahead...



Search locally for next best move



A neighbor node with lowest cost



Search locally for next best move

4	2	4
2	1	2
4	2	4

What should happen now ?

99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	8	6	4	6	90	99	99	99	99
99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	6	4	2	4	90	99	99	99	99
99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	4	2	[illegible]	2	90	99	99	99	99
99	90	90	[illegible]	90	90	90	90	90	90	90	90	90	90	99	90	6	4	2	4	90	99	99	99	99
99	90	84	[illegible]	80	78	76	74	72	70	68	66	64	90	99	90	8	6	4	6	90	90	99	99	99
99	90	82	80	78	76	74	72	70	68	66	64	62	90	99	90	10	8	6	8	10	90	99	99	99
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99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	99

Search locally for next best move

4	2	4
2	1	2
4	2	4

Stop search when no neighbor has a lower cost

99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	8	6	4	6	90	99	99	99	99
99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	6	4	2	4	90	99	99	99	99
99	99	99	99	99	99	99	99	99	99	99	99	99	99	99	90	4	2	[illegible]	2	90	99	99	99	99
99	90	90	[illegible]	90	90	90	90	90	90	90	90	90	90	99	90	6	4	2	4	90	99	99	99	99
99	90	84	[illegible]	80	78	76	74	72	70	68	66	64	90	99	90	8	6	4	6	90	90	99	99	99
99	90	82	80	78	76	74	72	70	68	66	64	62	90	99	90	10	8	6	8	10	90	99	99	99
99	90	84	82	80	78	76	90	90	90	90	62	60	90	99	90	12	10	8	10	12	90	99	99	99
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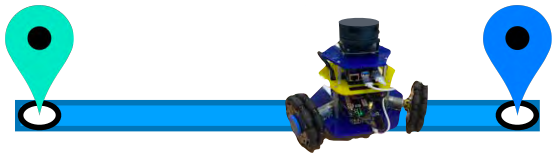
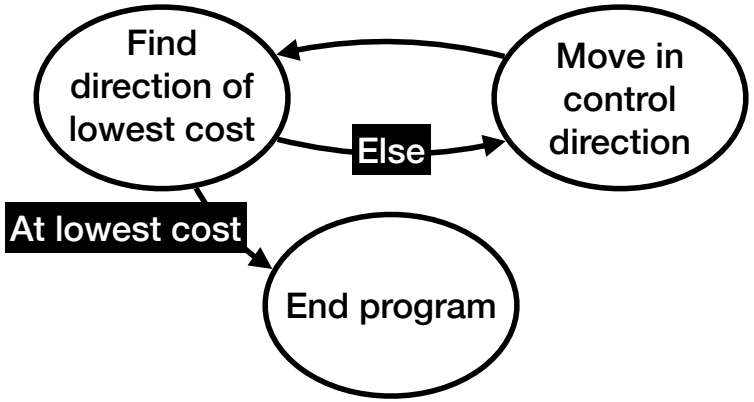


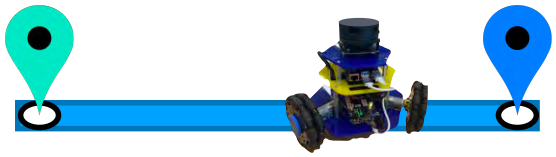
Just move randomly

Follow wall to goal

Build a map to guide us

A local search algorithm



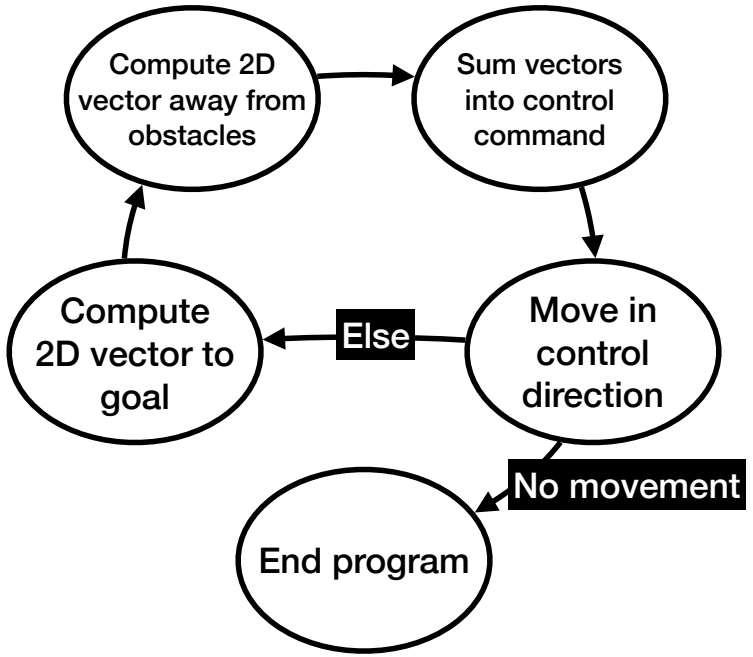


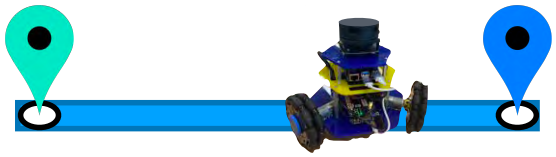
Just move randomly

Follow wall to goal

Build a map to guide us

Potential Field Navigation





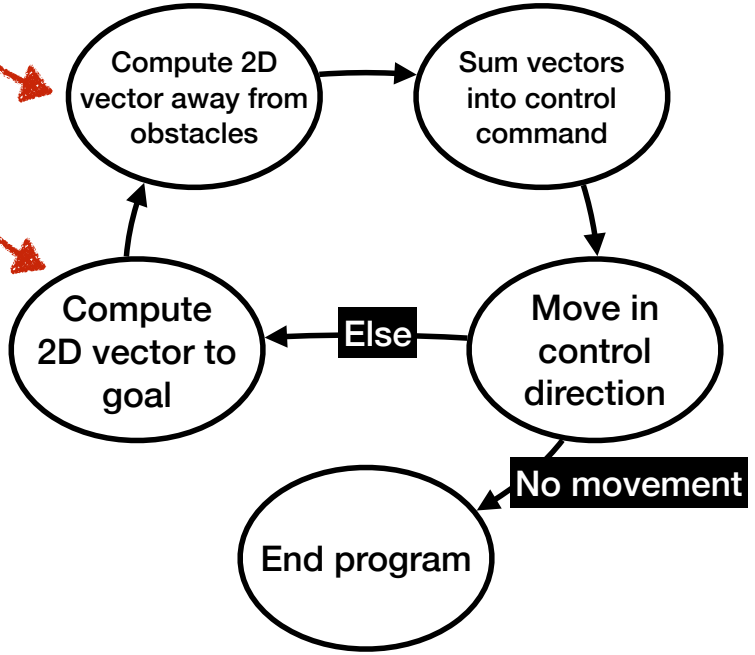
Just move randomly

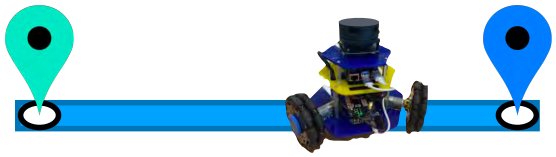
Follow wall to goal

Build a map to guide us

How do we do this ?

Potential Field Navigation





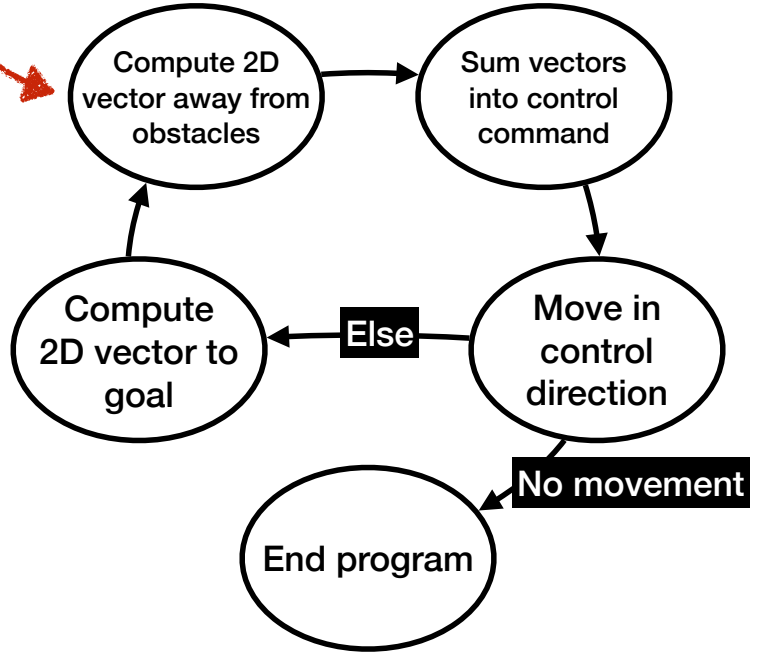
Just move randomly

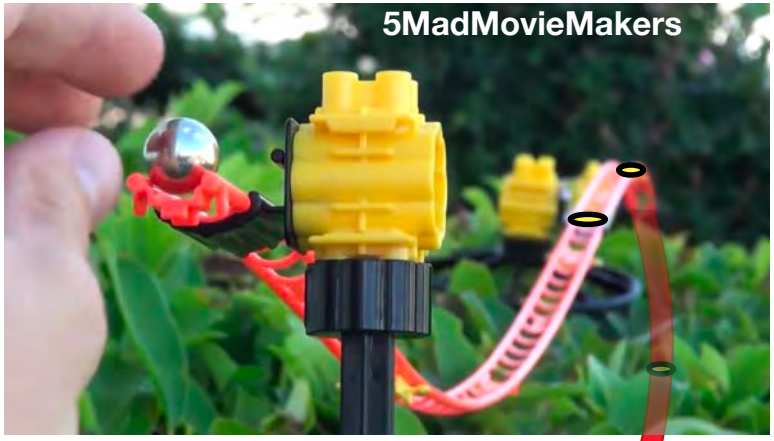
Follow wall to goal

Build a map to guide us

Upcoming: Distance Transform

Potential Field Navigation





Autonomous Navigation: Local Search

Robotics 102

Introduction to AI and Programming
University of Michigan and Berea College
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