

Potential Field Navigation: Creating Potentials

ROB 102: Introduction to AI & Programming

Lecture 08

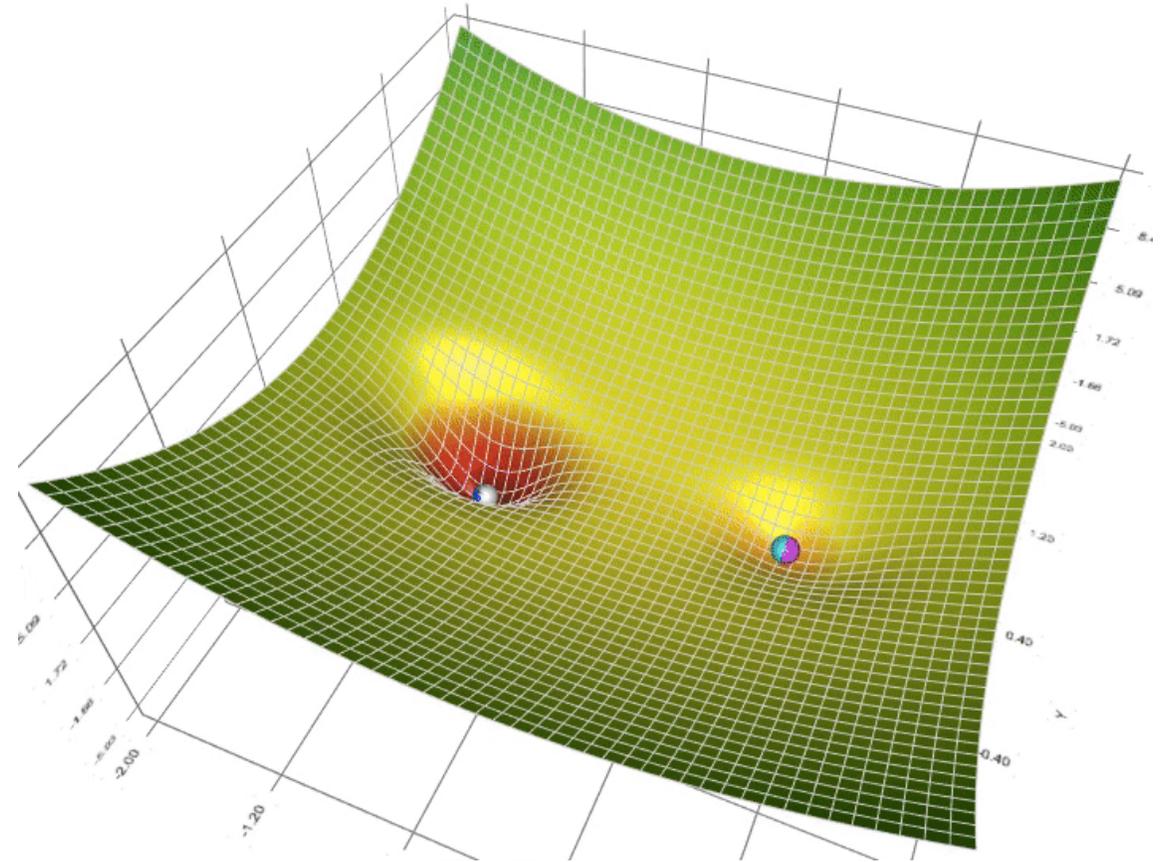
2021/10/20

Last time...

[\(Link to GIF\)](#)

A **potential field** has *high* value in areas the robot should avoid and *low* value where the robot should go.

The robot navigates by moving to the area in its local region with the lowest potential.



Last time...

An **attractive potential** pulls the robot towards a goal.

Attractive potential

$\sqrt{10}$	$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$
3	2	1	0	1
$\sqrt{10}$	$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$
$\sqrt{13}$	$\sqrt{8}$	$\sqrt{5}$	2	$\sqrt{5}$
$\sqrt{18}$	$\sqrt{13}$	$\sqrt{10}$	3	$\sqrt{10}$

Last time...

An **attractive potential** pulls the robot towards a goal.

A **distance transform** gives the distance to the nearest occupied cell for each cell.

$\sqrt{10}$	$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$
3	2	1	0	1
$\sqrt{10}$	$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$
$\sqrt{13}$	$\sqrt{8}$	$\sqrt{5}$	2	$\sqrt{5}$
$\sqrt{18}$	$\sqrt{13}$	$\sqrt{10}$	3	$\sqrt{10}$

Attractive potential

2	1	2	2	1	0
1	0	1	2	1	0
1	0	1	2	2	1
1	0	0	1	2	2
1	0	0	1	2	3
2	1	1	2	3	4

Manhattan
distance transform

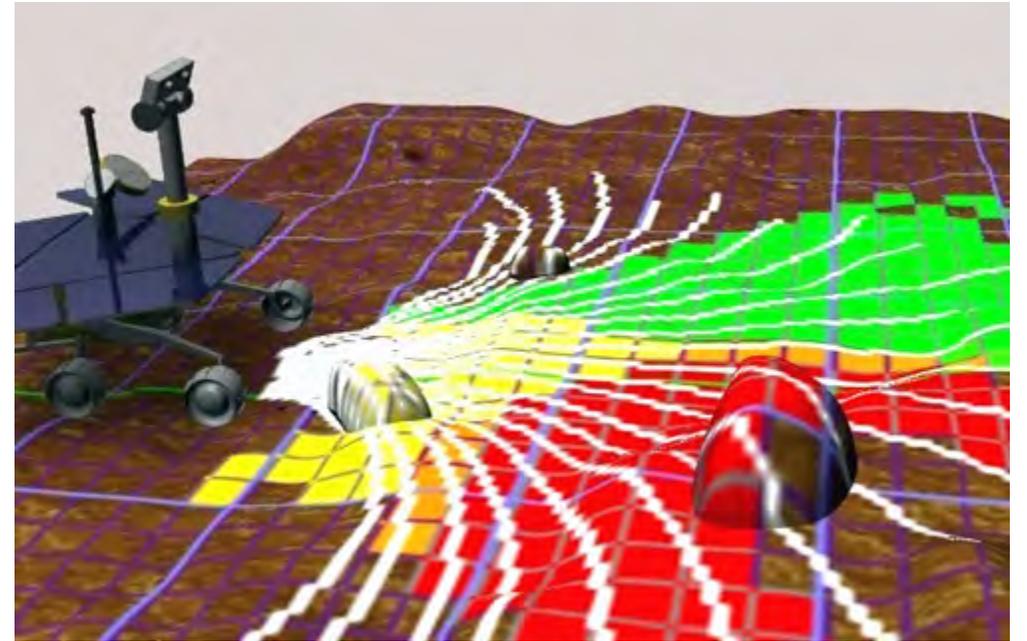
Potential Fields in the Wild

A rover can use potential field navigation to drive over rough terrain.

Potential field control is used for **local navigation**, to nearby goals.

What about global navigation?

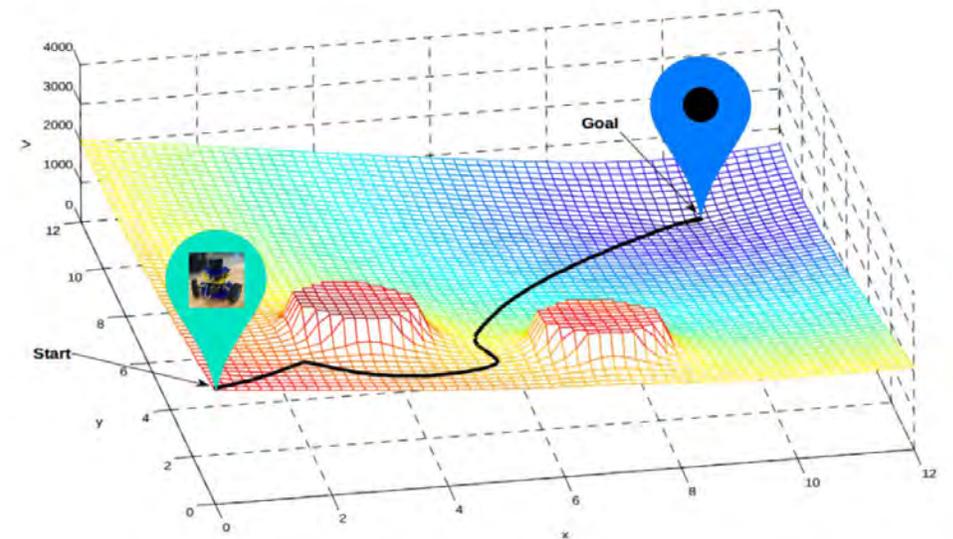
Global navigation: From any start to any goal, in any map



Project 2: Potential Field Navigation

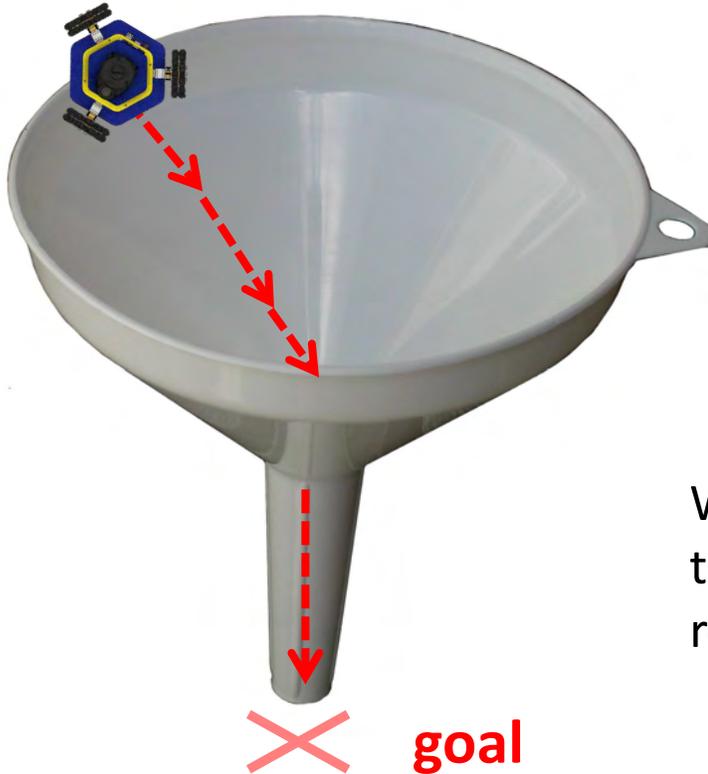
- Build a map of environment
- Form attraction potential to goal
- Form repulsion potentials away from obstacles
- Add potentials together into potential field
- Local search over potential field to navigate

This lecture {



Attraction Potential & Local Search

An **attractive potential** *pulls* us towards a goal location.

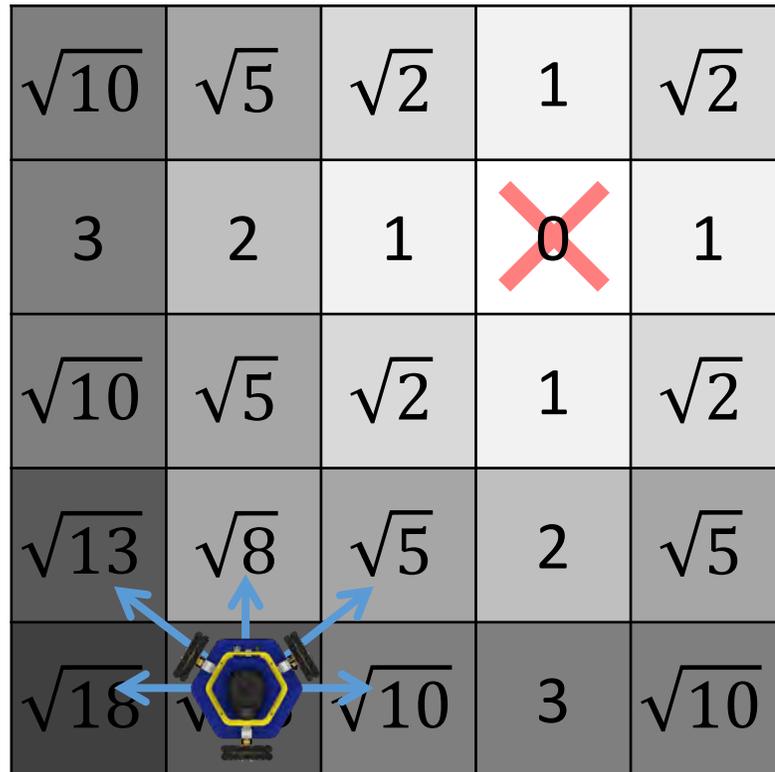


We can think of the potential as the **potential energy** pulling the robot down a hill.

Attraction Potential & Local Search

On a map, the **distance to the goal cell** is a useful attractive potential.

$\sqrt{10}$	$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$
3	2	1	0	1
$\sqrt{10}$	$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$
$\sqrt{13}$	$\sqrt{8}$	$\sqrt{5}$	2	$\sqrt{5}$
$\sqrt{18}$	$\sqrt{9}$	$\sqrt{10}$	3	$\sqrt{10}$



Attraction Potential & Local Search

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3	2	1	0	1
$\sqrt{10}$	$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$
$\sqrt{13}$	$\sqrt{8}$		2	$\sqrt{5}$
$\sqrt{18}$	$\sqrt{13}$	$\sqrt{10}$	3	$\sqrt{10}$



Attraction Potential & Local Search

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$\sqrt{10}$	$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$
$\sqrt{13}$	$\sqrt{8}$		2	$\sqrt{5}$
$\sqrt{18}$	$\sqrt{13}$	$\sqrt{10}$	3	$\sqrt{10}$

The table shows a 5x5 grid of cells. The central cell (row 4, column 3) contains a robot icon. Blue arrows point from the robot to its eight adjacent cells. The values in the cells represent the distance to the goal cell (row 2, column 4), which is marked with a red 'X' and a '0'. The values are: $\sqrt{10}$, $\sqrt{5}$, $\sqrt{2}$, 1, $\sqrt{2}$ (top row); 3, 2, 1, ~~0~~, 1 (second row); $\sqrt{10}$, $\sqrt{5}$, $\sqrt{2}$, 1, $\sqrt{2}$ (third row); $\sqrt{13}$, $\sqrt{8}$, $\sqrt{10}$, 2, $\sqrt{5}$ (fourth row); $\sqrt{18}$, $\sqrt{13}$, $\sqrt{10}$, 3, $\sqrt{10}$ (bottom row).



Attraction Potential & Local Search

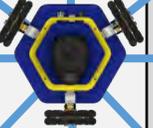
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Attraction Potential & Local Search

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$\sqrt{13}$	$\sqrt{8}$	$\sqrt{5}$	2	$\sqrt{5}$
$\sqrt{18}$	$\sqrt{13}$	$\sqrt{10}$	3	$\sqrt{10}$

The table shows a 5x5 grid of cells. The central cell (row 3, column 4) contains a robot icon and the value 0, which is crossed out with a red 'X'. Blue arrows point from the robot to its four immediate neighbors (up, down, left, right). The values in the grid represent the distance from the robot to the goal cell, which is located at the center of the grid (row 2, column 4).



Attraction Potential & Local Search

On a map, the **distance to the goal cell** is a useful attractive potential.

$\sqrt{10}$	$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$
3	2	1		1
$\sqrt{10}$	$\sqrt{5}$	$\sqrt{2}$	1	$\sqrt{2}$
$\sqrt{13}$	$\sqrt{8}$	$\sqrt{5}$	2	$\sqrt{5}$
$\sqrt{18}$	$\sqrt{13}$	$\sqrt{10}$	3	$\sqrt{10}$



The Attraction Potential

Let's define two vectors:

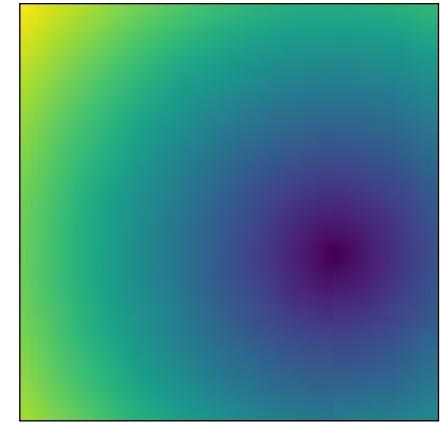
`goal_dists` The distance from each cell to the goal cell.

`attract_field` The attraction potential at each cell.

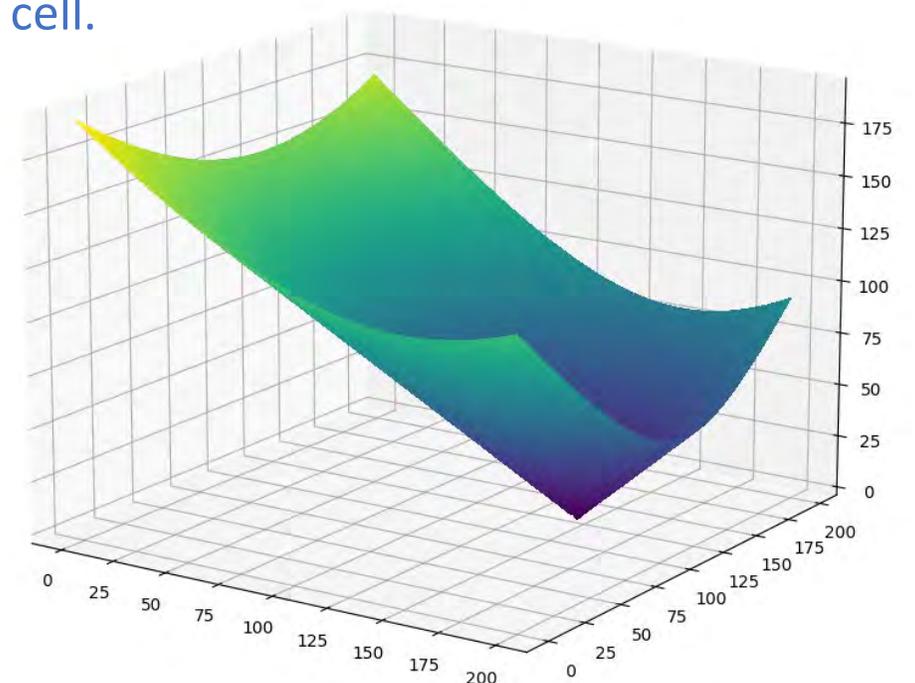
Let's define the potential at cell i as:

```
attract_field[i] = goal_dists[i]
```

This defines a **cone potential**.



Cone potential in 2D



The Attraction Potential

Let's define two vectors:

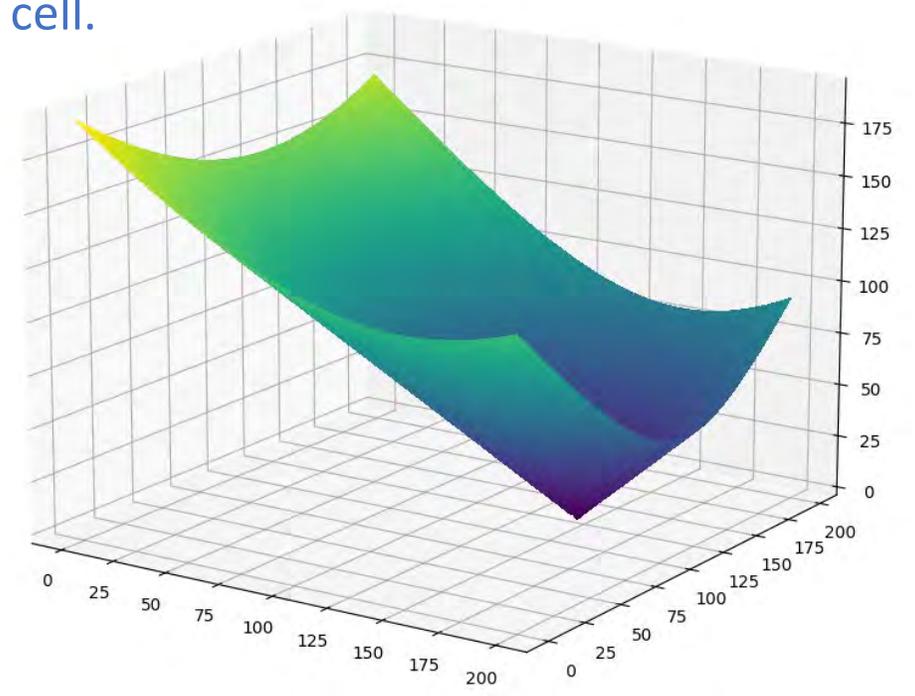
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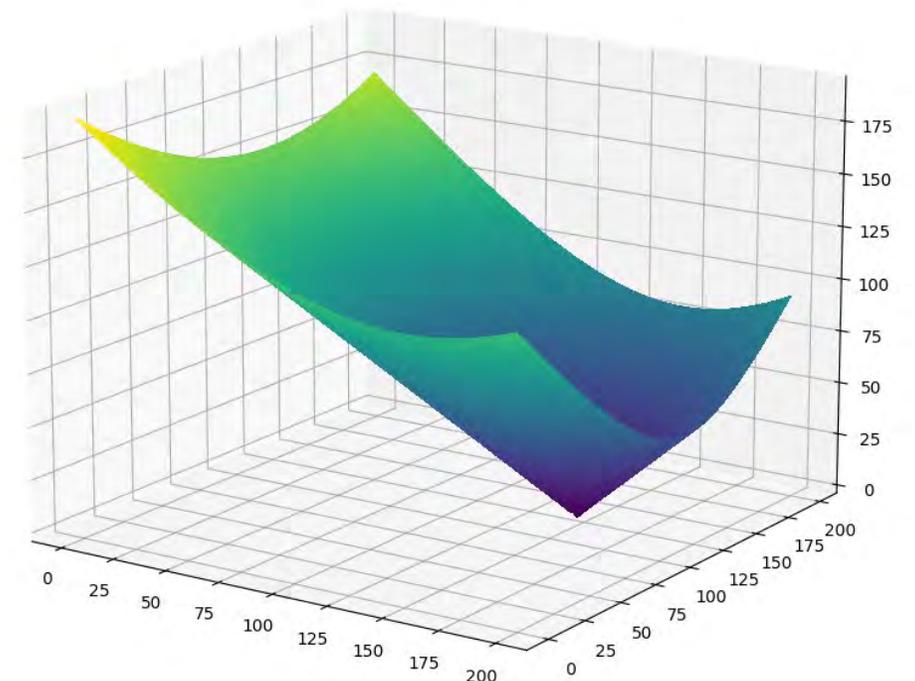


The Attraction Potential

The cone potential:

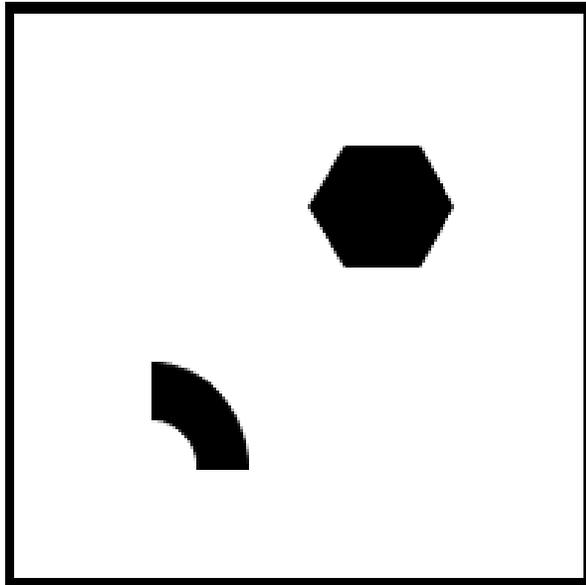
```
attract_field[i] = goal_dists[i]
```

Problem: Doesn't avoid obstacles.

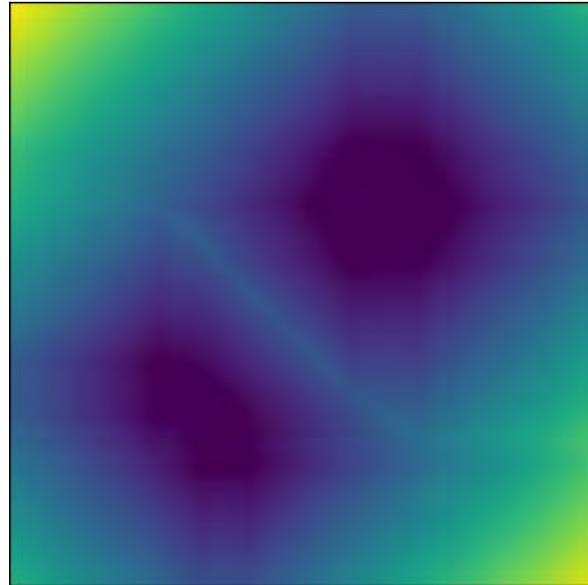


The Distance Transform

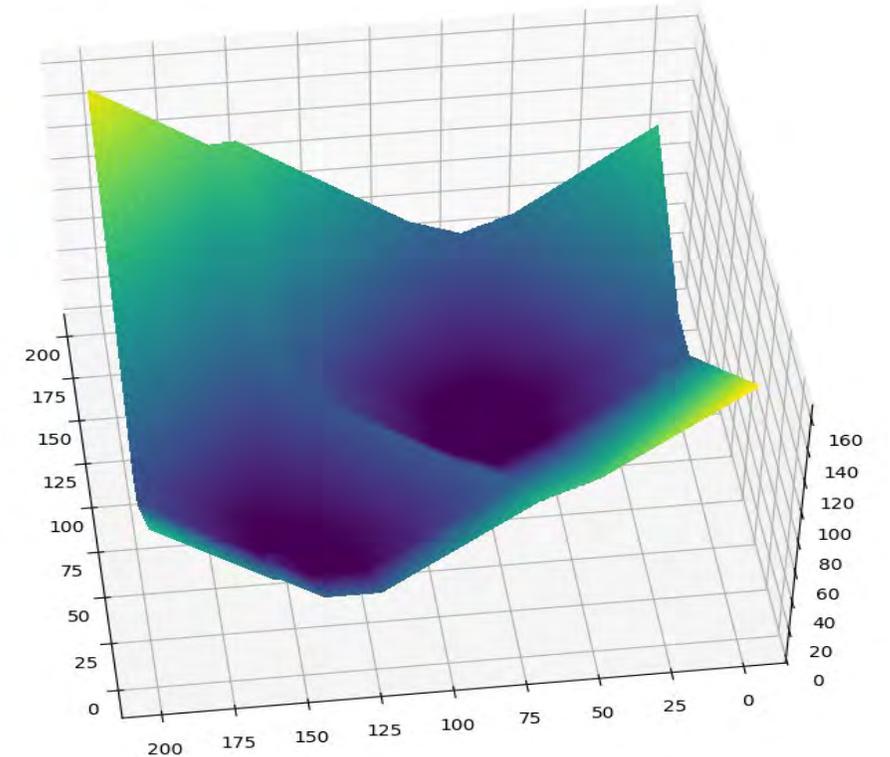
The **distance transform** gives the distance to the nearest obstacle at each cell.



Binary Image



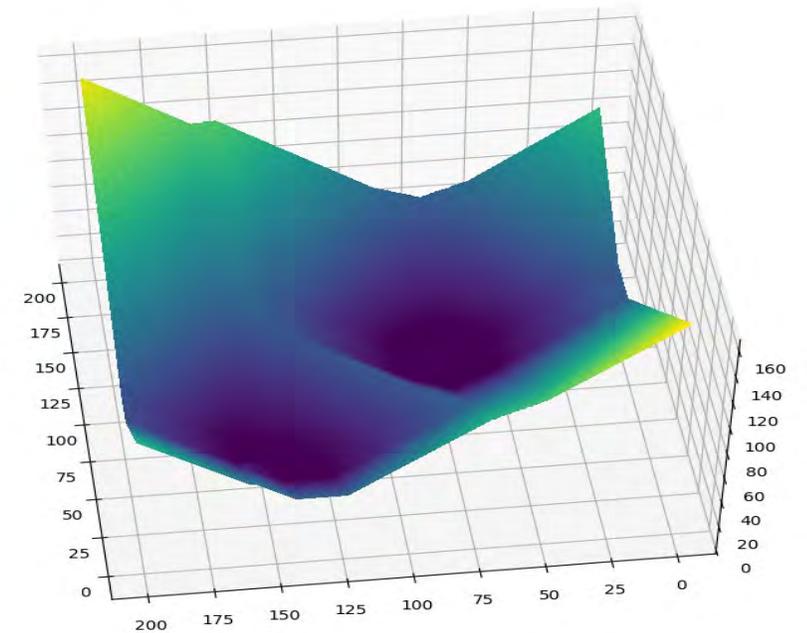
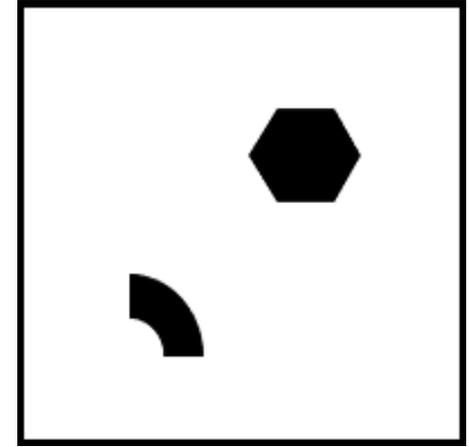
2D Distance Transform



The Distance Transform

The distance transform is **LOW** where the robot should avoid and **HIGH** where the robot should go.

This is the opposite of the repulsive field.



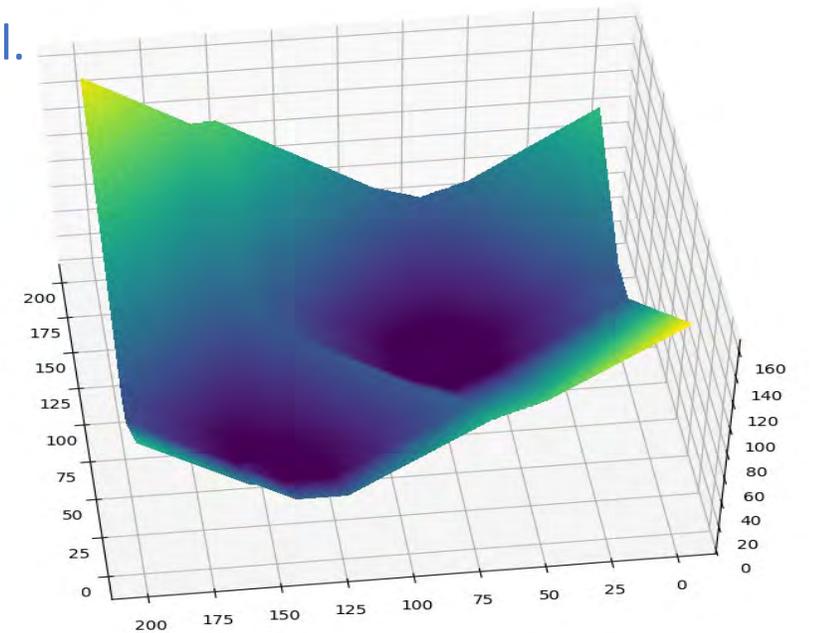
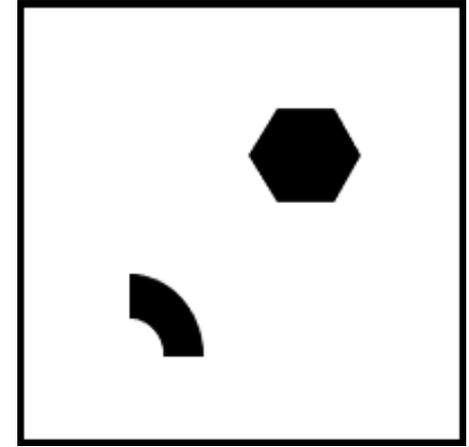
Distance Transform

The Repulsion Potential

Let's define two vectors:

`obstacle_dists` The distance from each cell to the nearest obstacle.

`repul_field` The repulsion potential at each cell.



Distance Transform

The Repulsion Potential

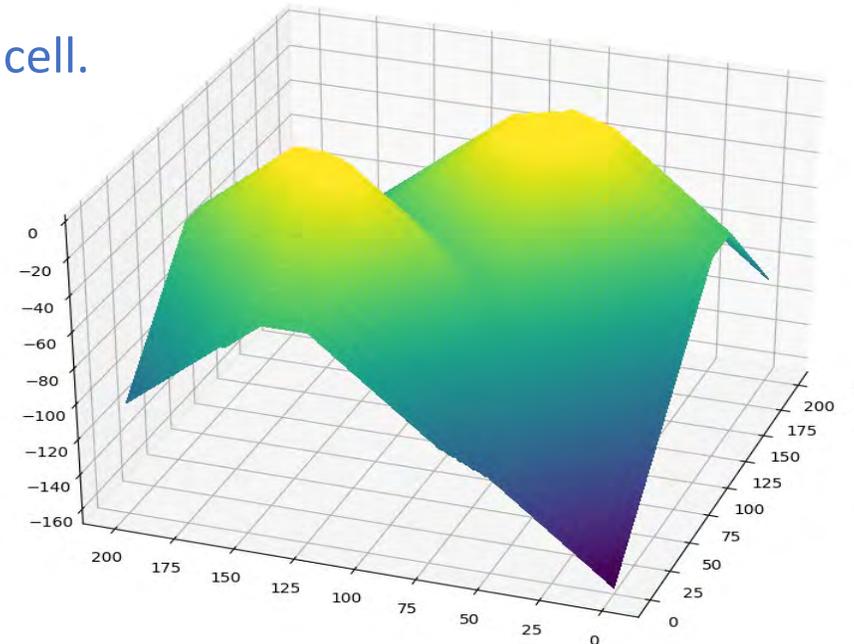
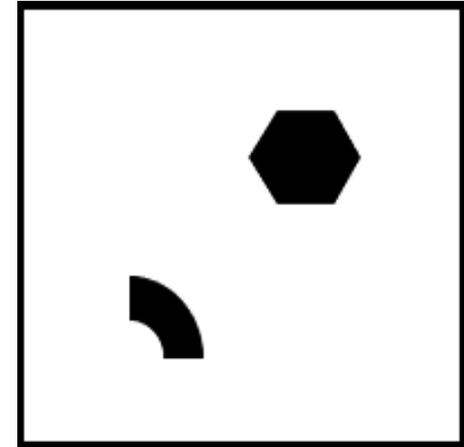
Let's define two vectors:

`obstacle_dists` The distance from each cell to the nearest obstacle.

`repul_field` The repulsion potential at each cell.

Idea: The repulsion field is the negative of the distance transform.

```
repul_field[i] = -obstacle_dists[i]
```



Repulsion Potential

Combining Potential Fields

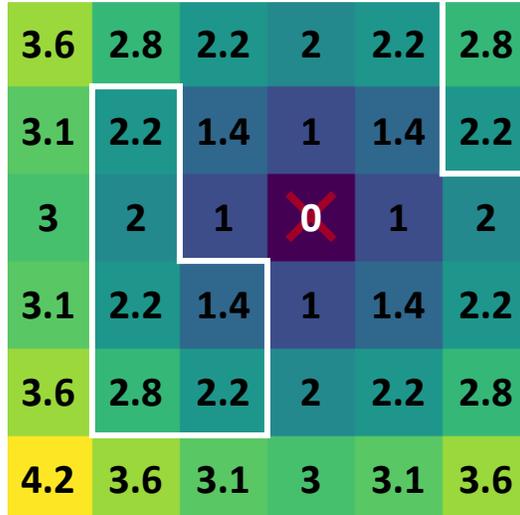
Idea: The potential field is the attractive field plus the repulsive field.

$$\text{potential}[i] = \text{attract_field}[i] + \text{repul_field}[i]$$

Combining Potential Fields

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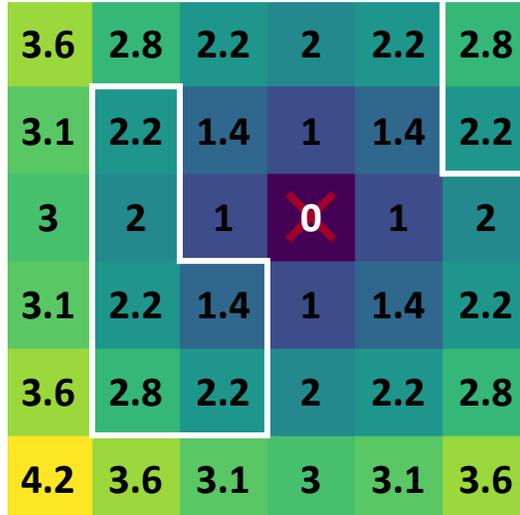


`attract_field = goal_dists`

Combining Potential Fields

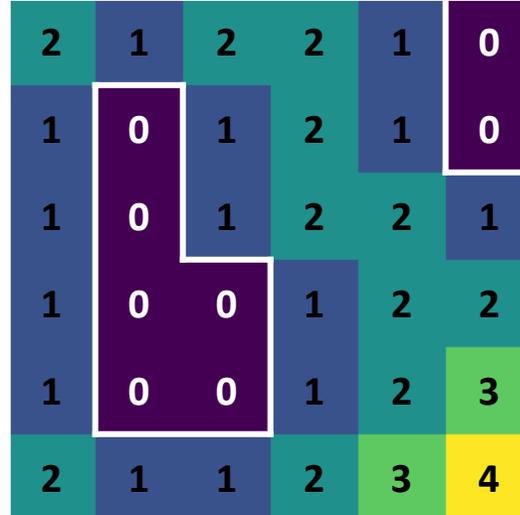
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$$\text{potential}[i] = \text{attract_field}[i] + \text{repul_field}[i]$$



attract_field

+ -1*

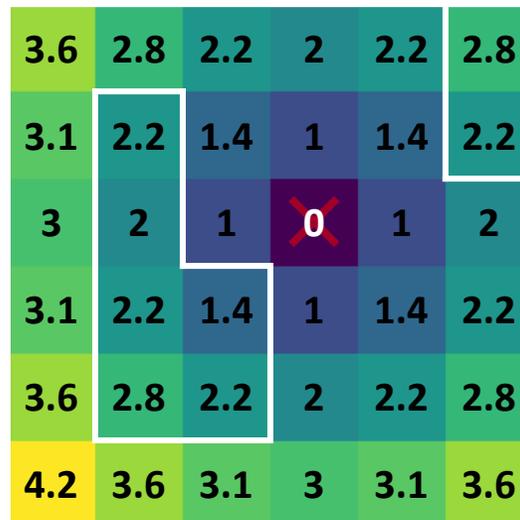


repul_field = -obstacle_dists

Combining Potential Fields

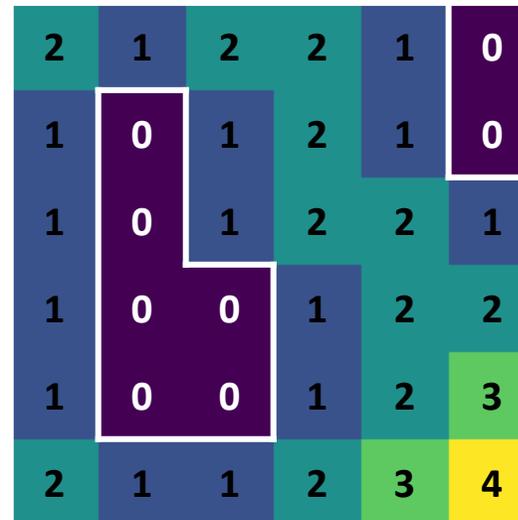
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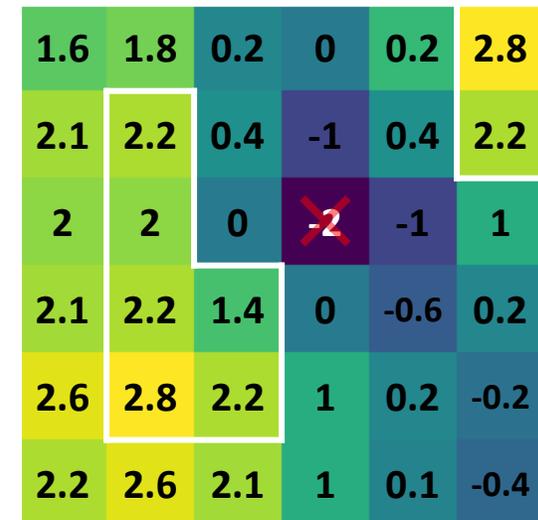
attract_field

+ -1*



repul_field

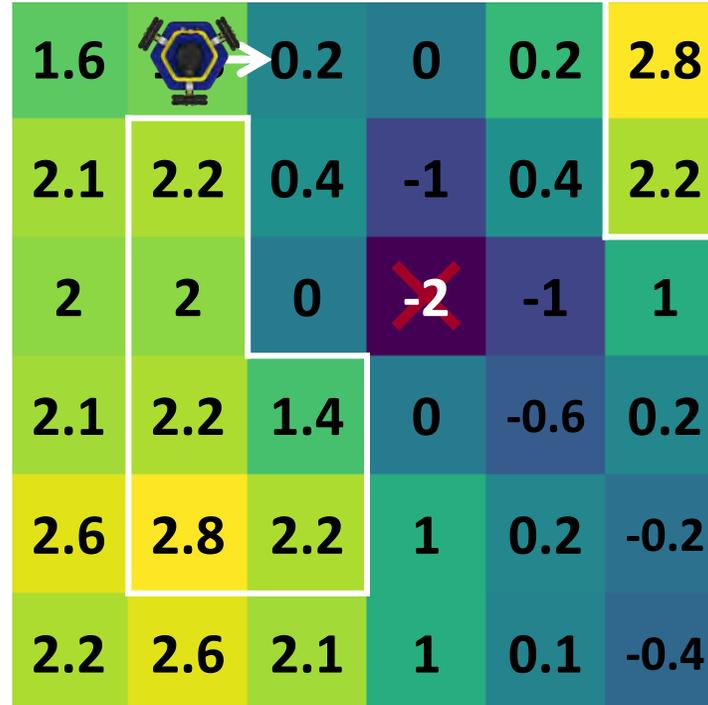
=



potential

Combining Potential Fields

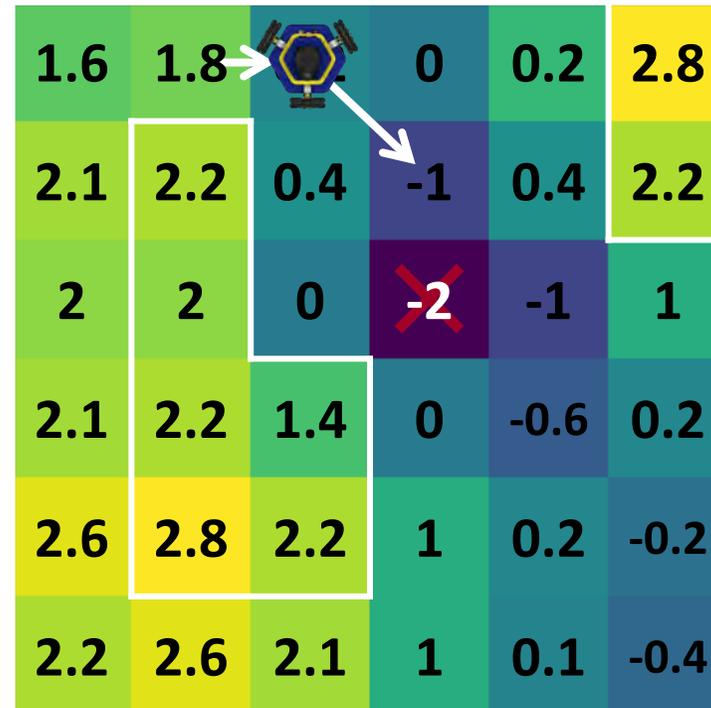
Let's take a closer look at our potential field.



potential

Combining Potential Fields

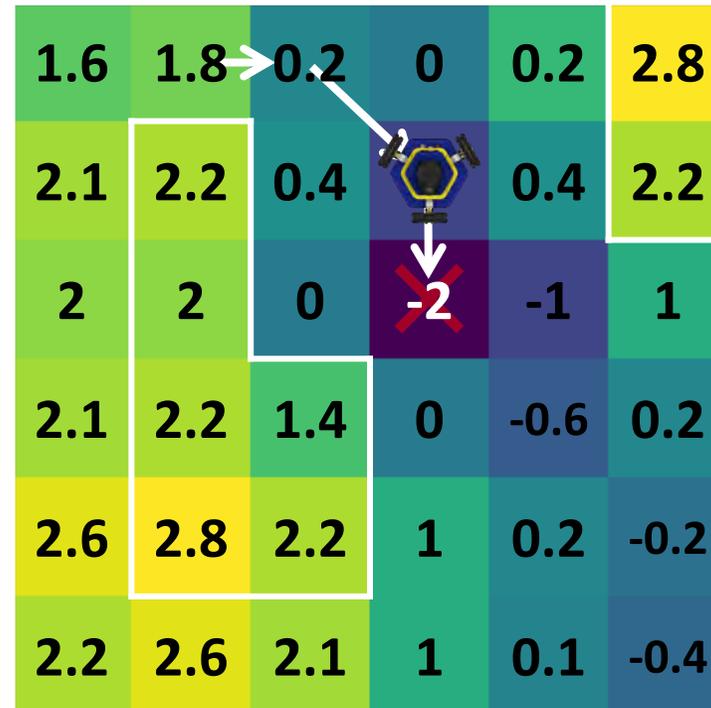
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potential

Combining Potential Fields

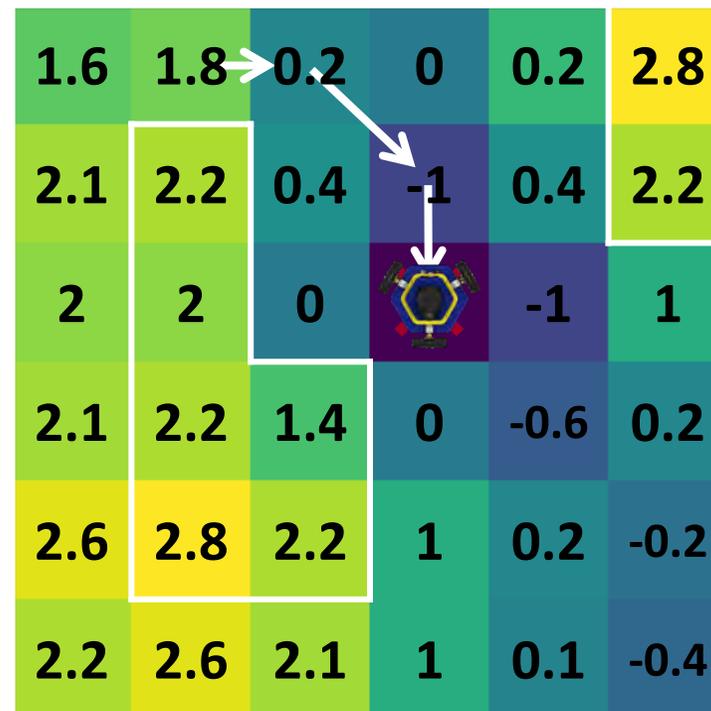
Let's take a closer look at our potential field.



potential

Combining Potential Fields

Let's take a closer look at our potential field.

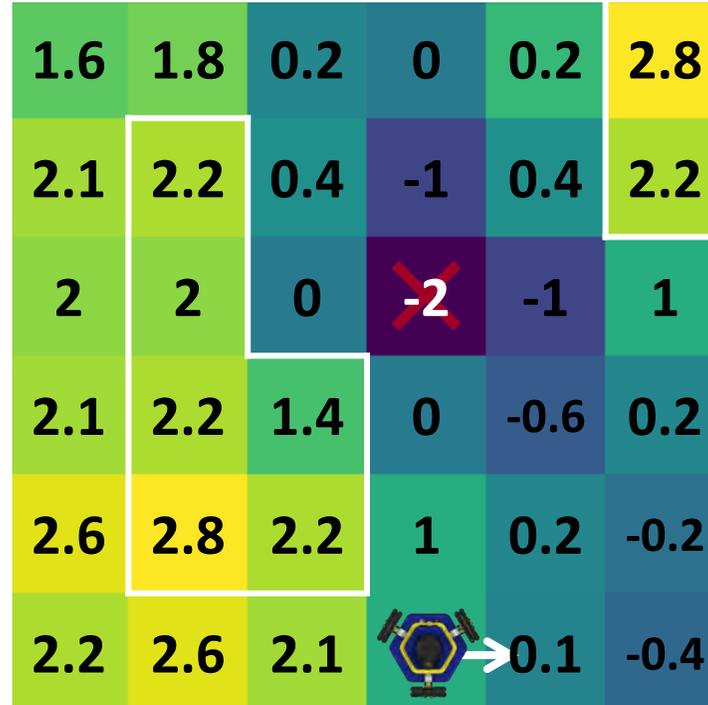


Success!

potential

Combining Potential Fields

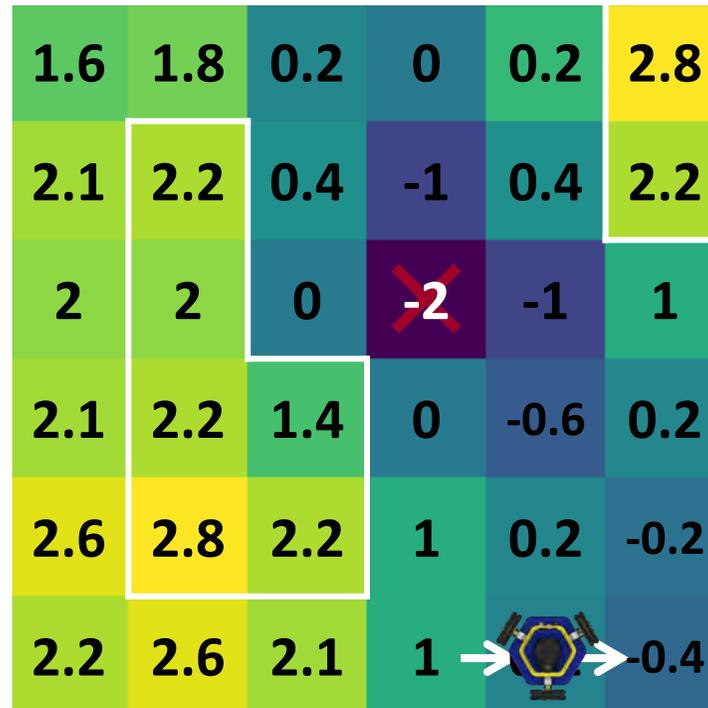
Let's take a closer look at our potential field.



potential

Combining Potential Fields

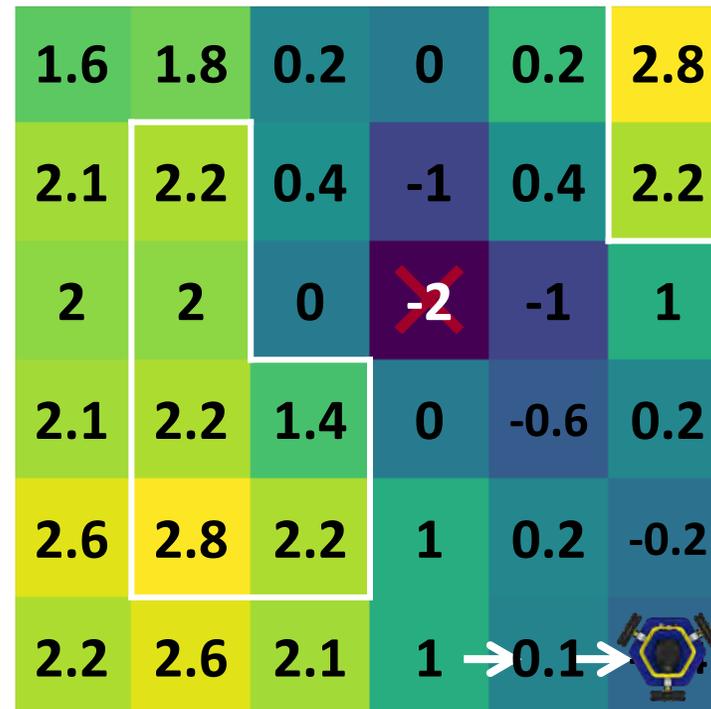
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potential

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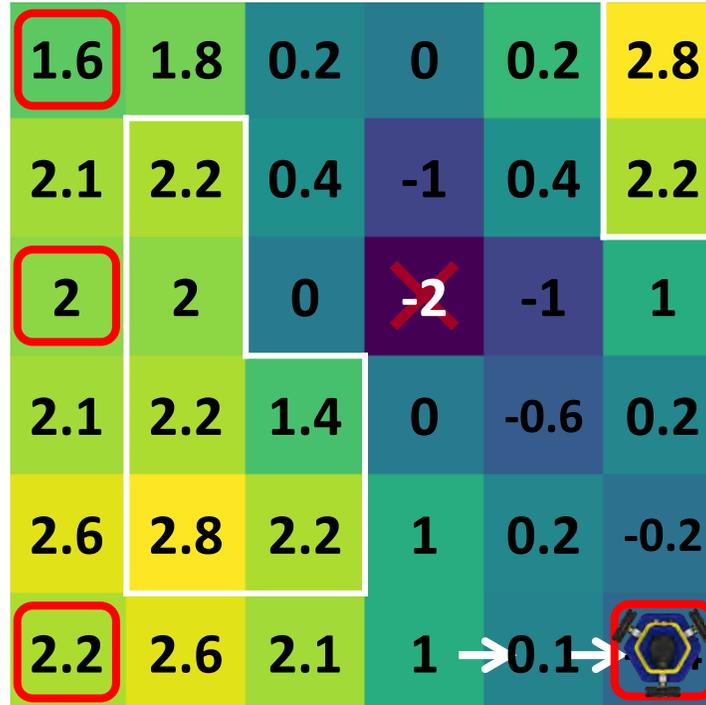


Local Minimum :(

potential

Combining Potential Fields

Let's take a closer look at our potential field.

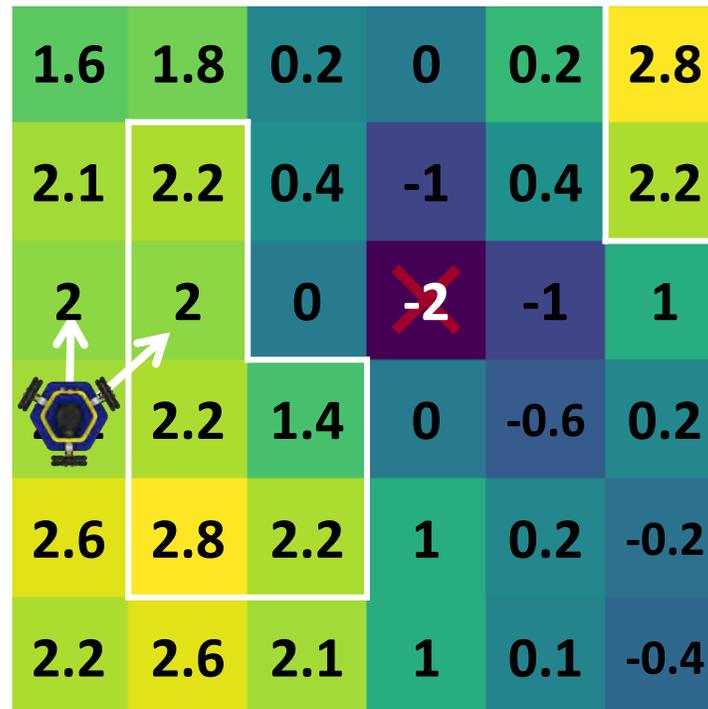


Lots of local minima!

potential

Combining Potential Fields

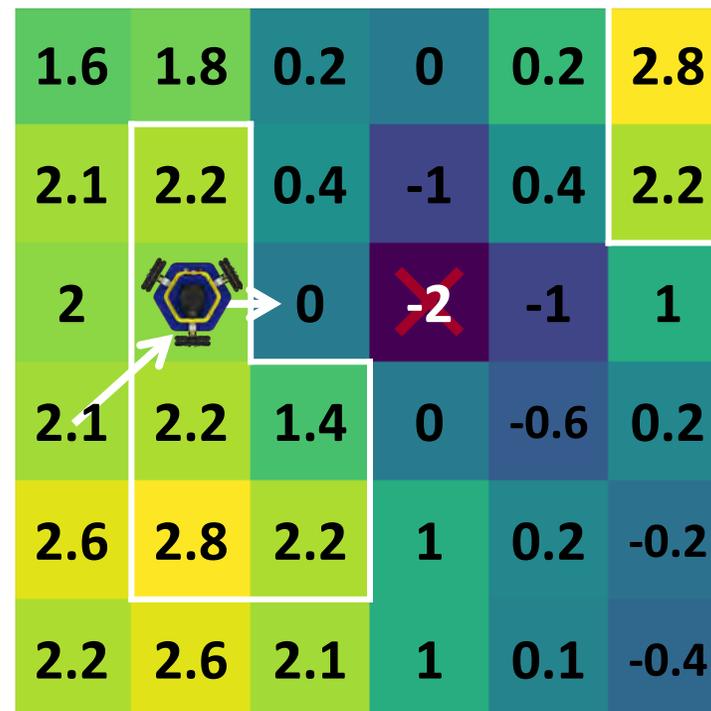
Let's take a closer look at our potential field.



potential

Combining Potential Fields

Let's take a closer look at our potential field.

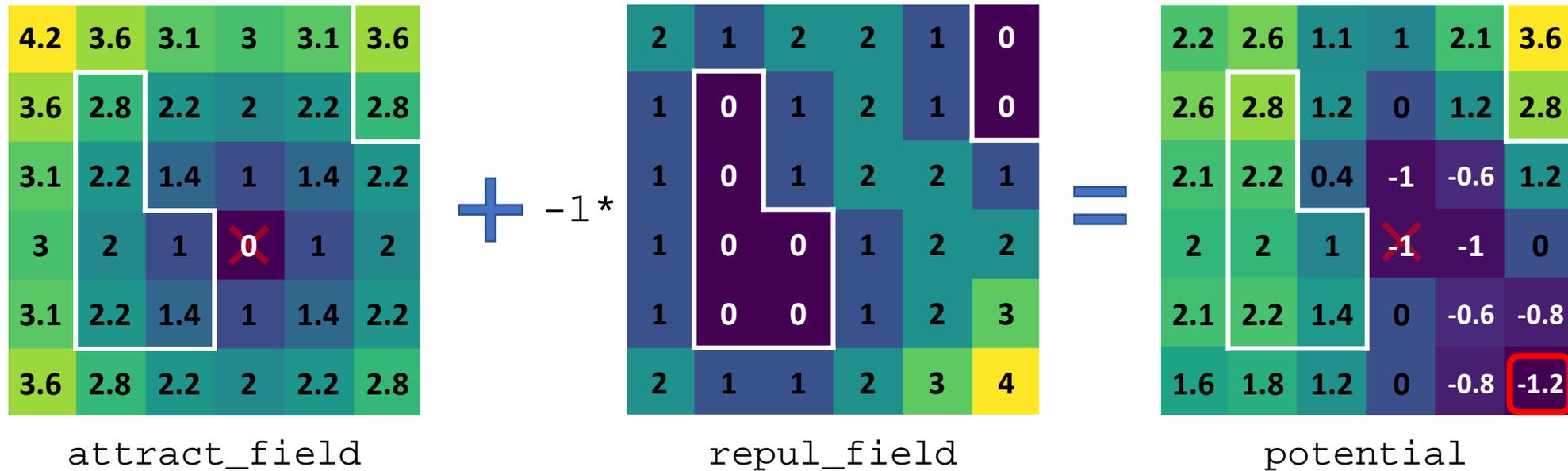


Collision!!

potential

Combining Potential Fields

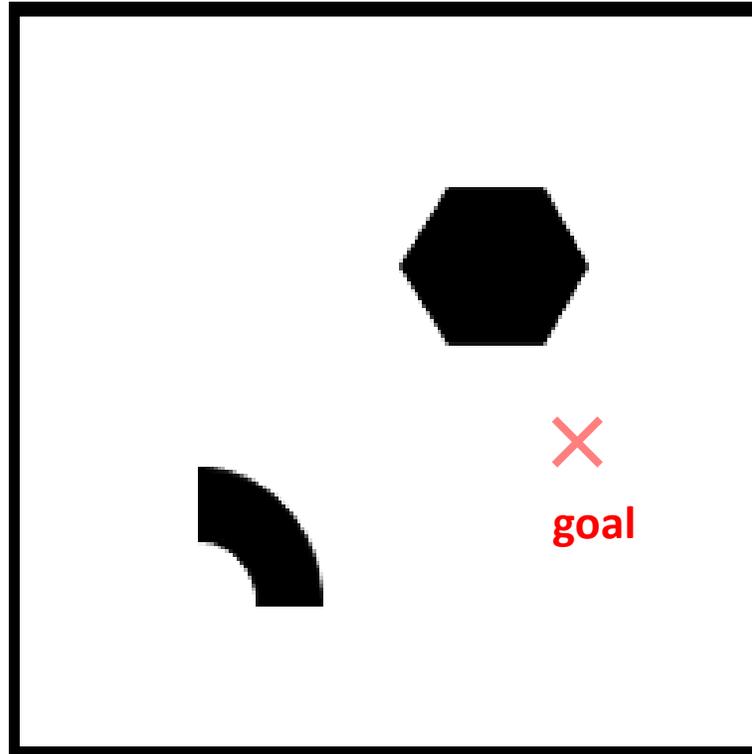
Depending on the goal, the lowest point of the potential field might not be at the goal location!



Computing the Potential Field

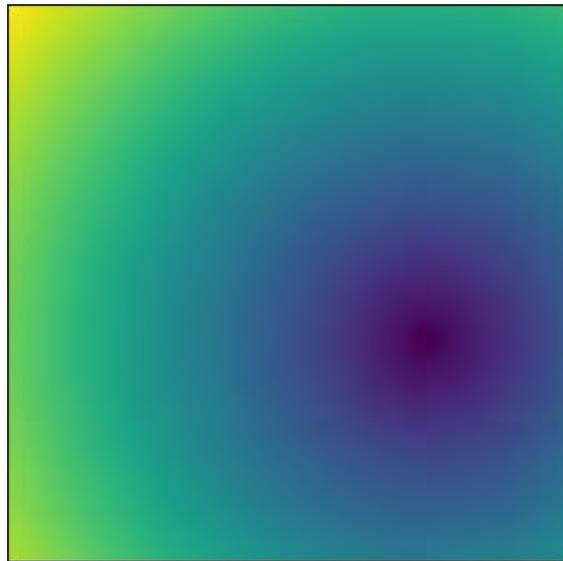
A larger example:

two_obstacles.map



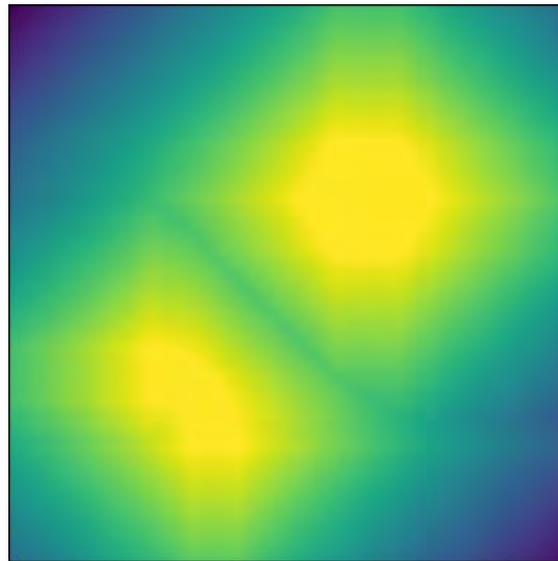
Computing the Potential Field

A larger example:



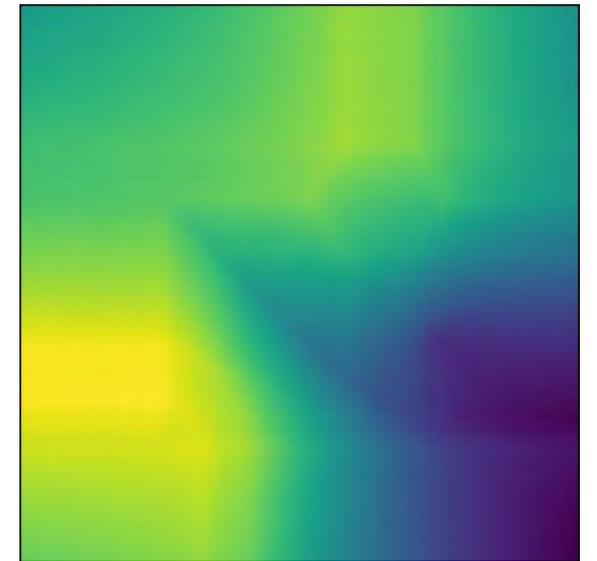
attract_field

+

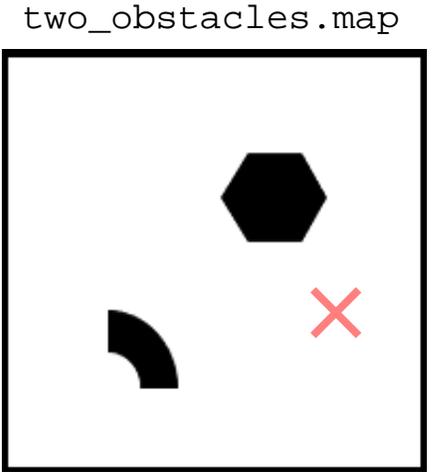


repul_field

=

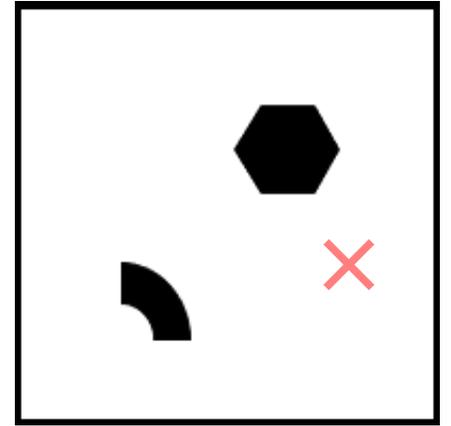


potential

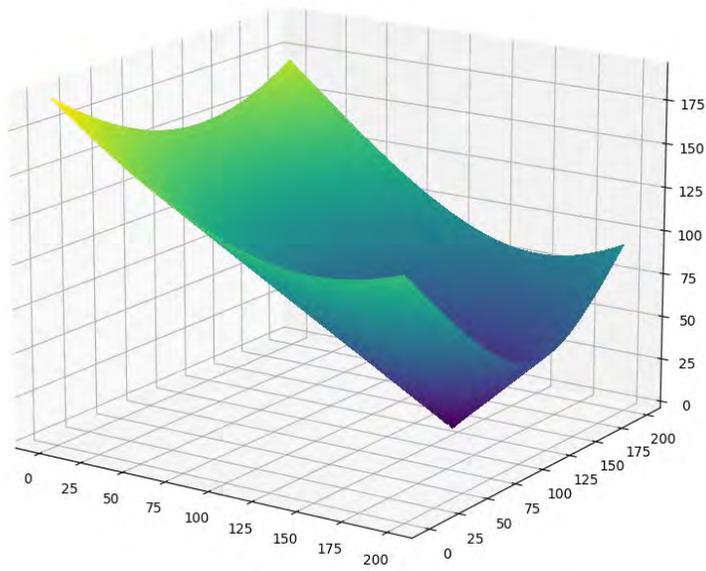


Computing the Potential Field

two_obstacles.map

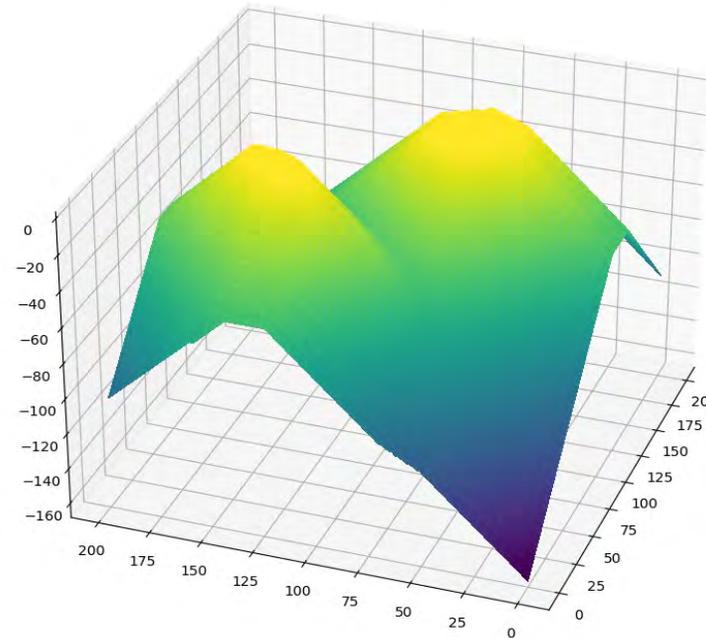


A larger example:



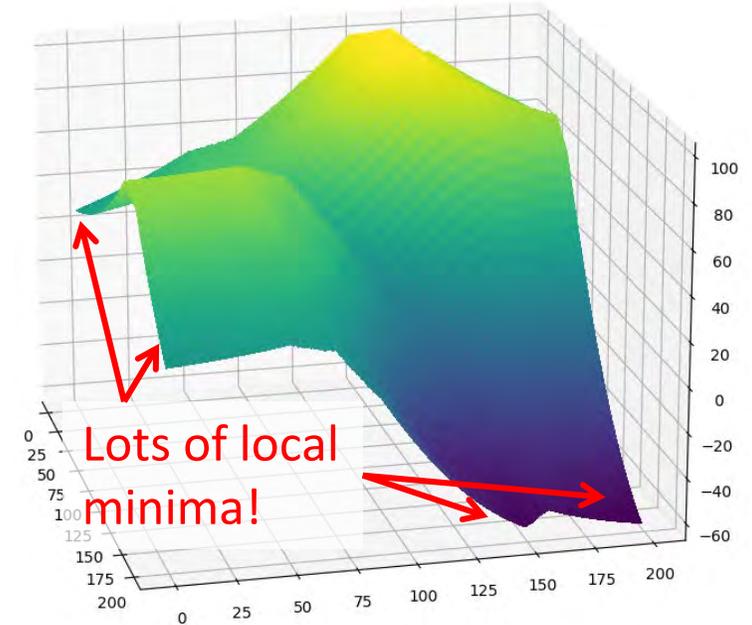
attract_field

+



repul_field

=



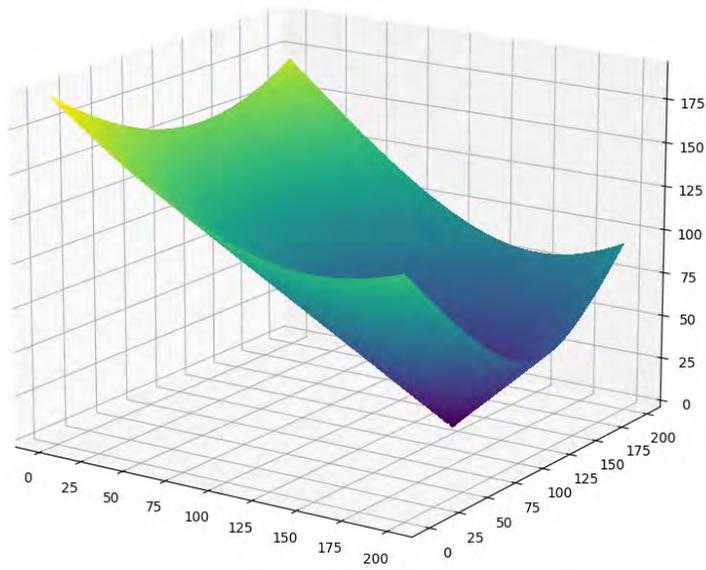
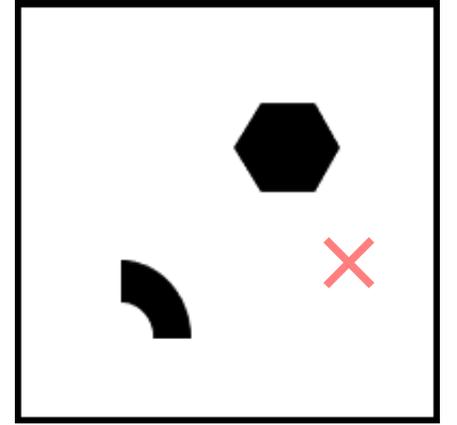
Lots of local minima!

potential

Computing the Potential Field

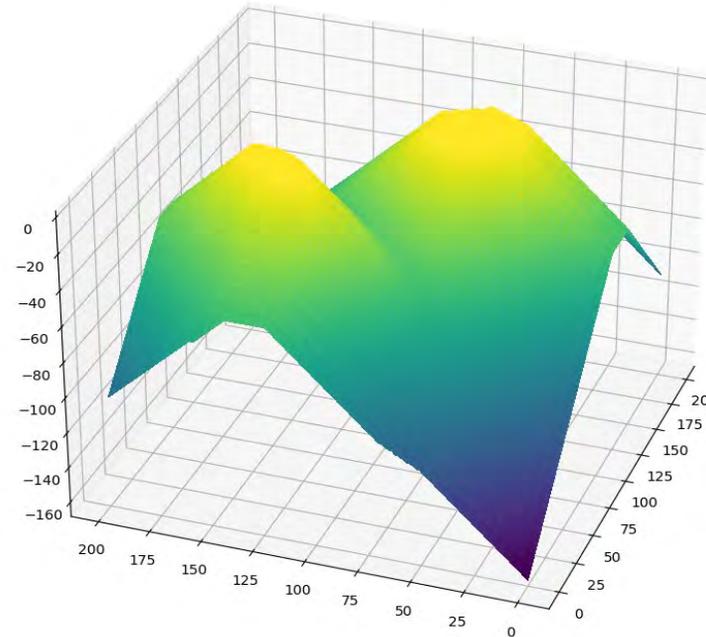
Can we do better?

two_obstacles.map



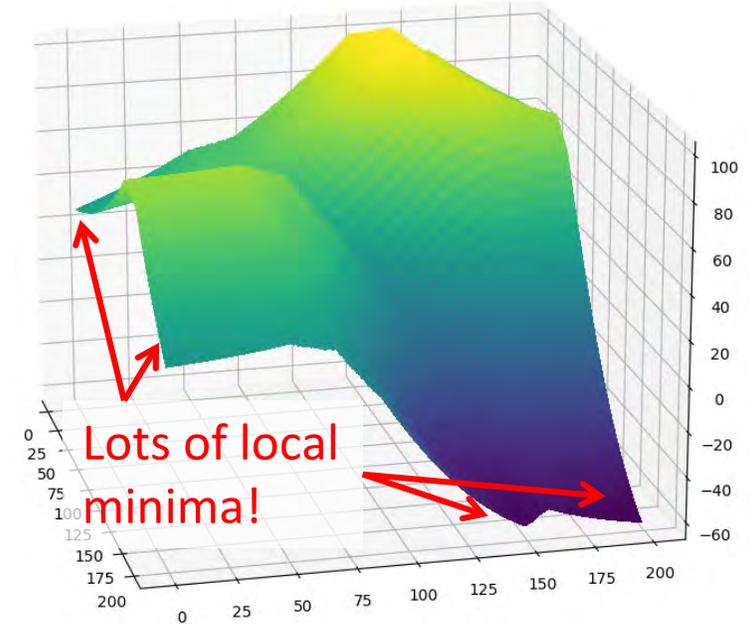
attract_field

+



repul_field

=

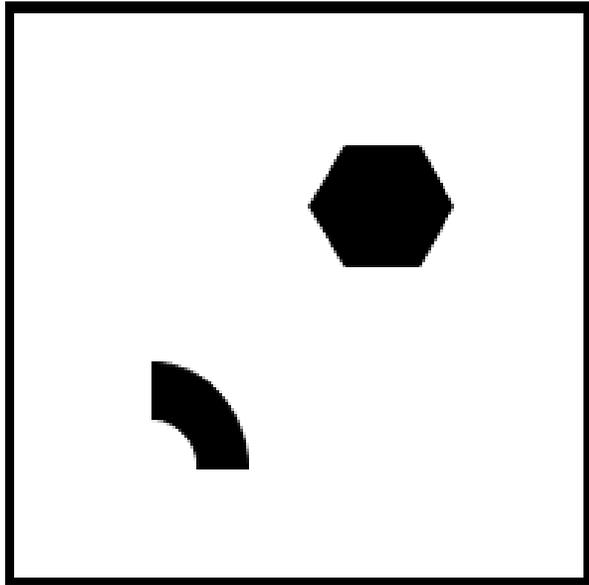


potential

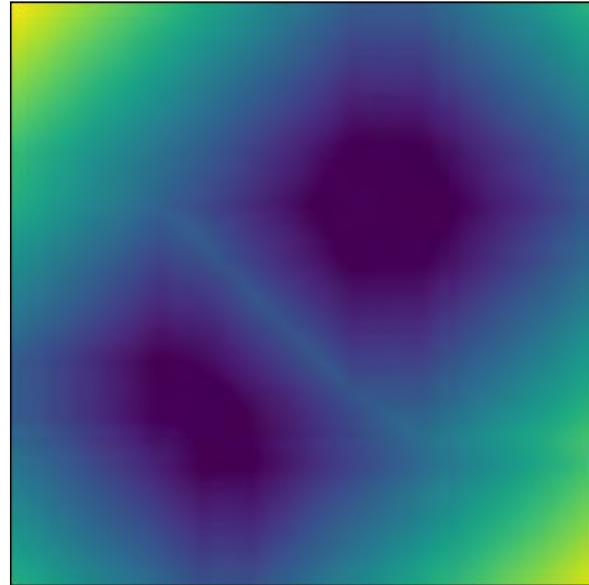
The Repulsion Potential

Idea: Apply a function to the distances to control the shape of the field.

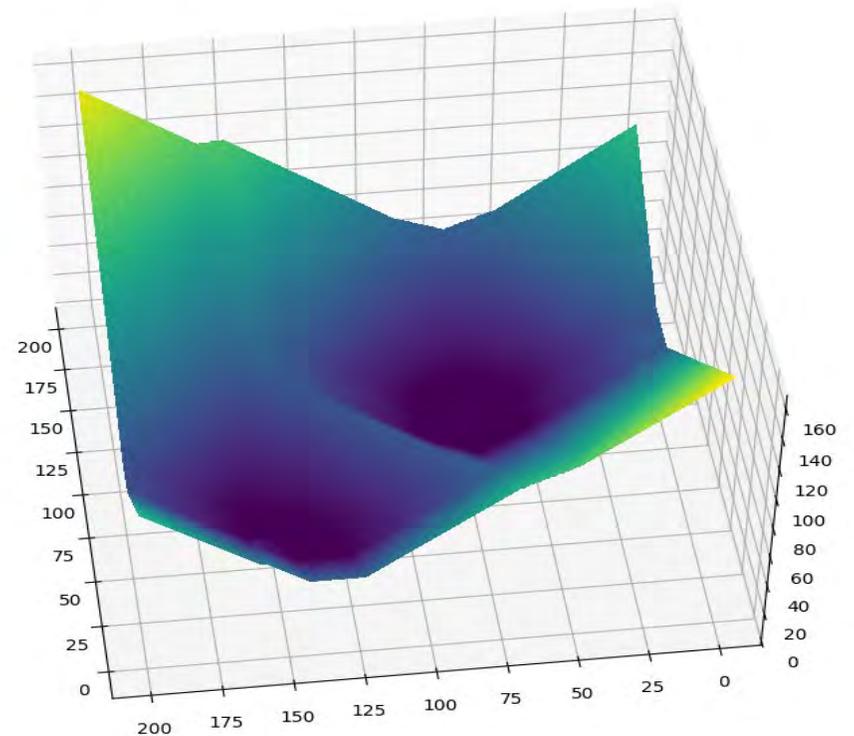
$$\text{repul_field}[i] = f(\text{obstacle_dists}[i])$$



Binary Image



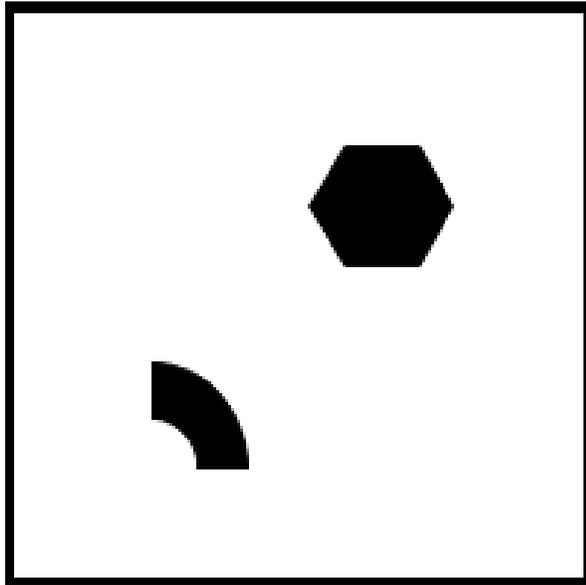
obstacle_dists



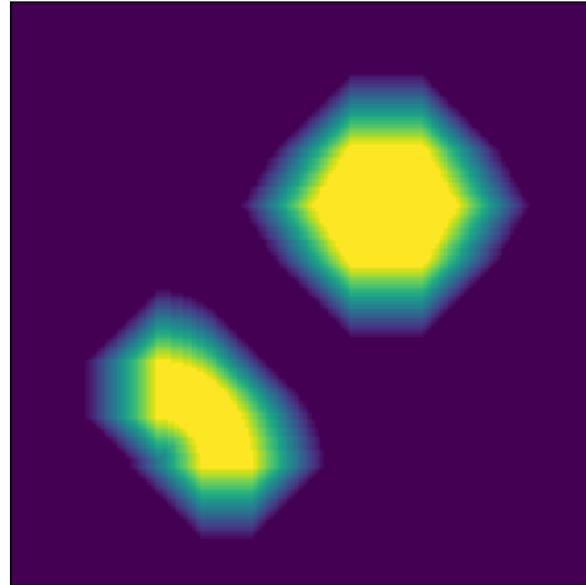
The Repulsion Potential

Idea: Threshold the obstacle distances then take the negative.

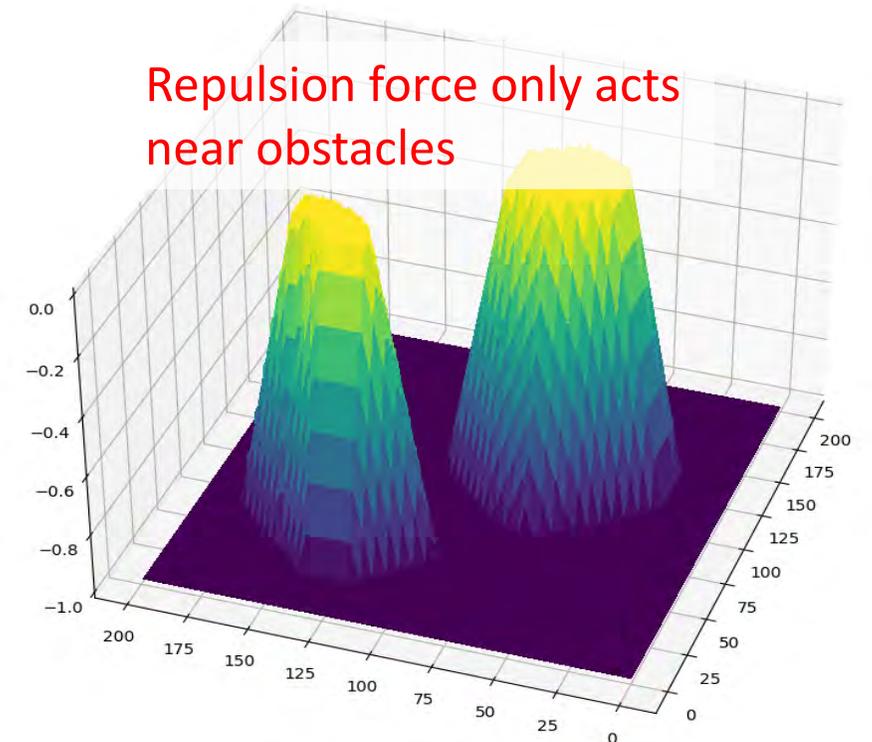
```
repul_field[i] = -min(obstacle_dists[i], THRESH)
```



Binary Image



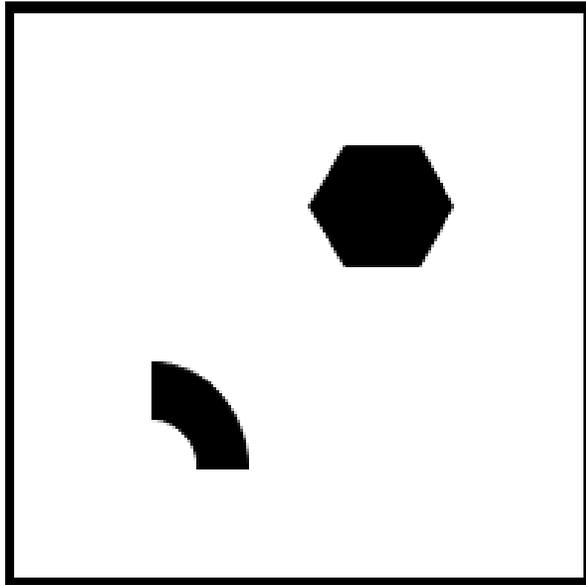
repul_field



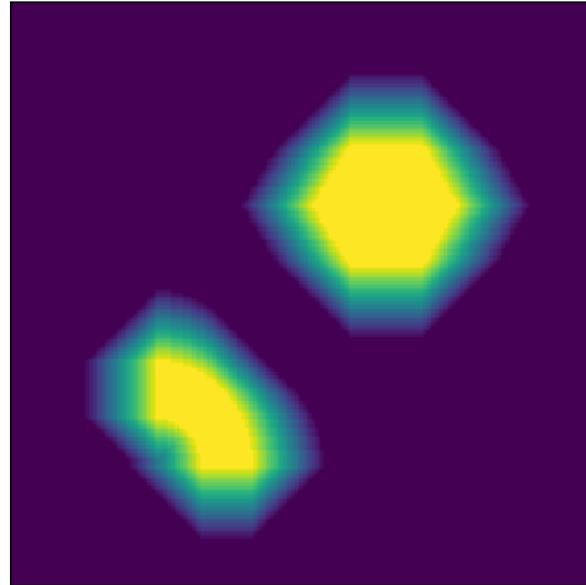
The Repulsion Potential

Idea: Normalize the potential so its range doesn't depend on threshold.

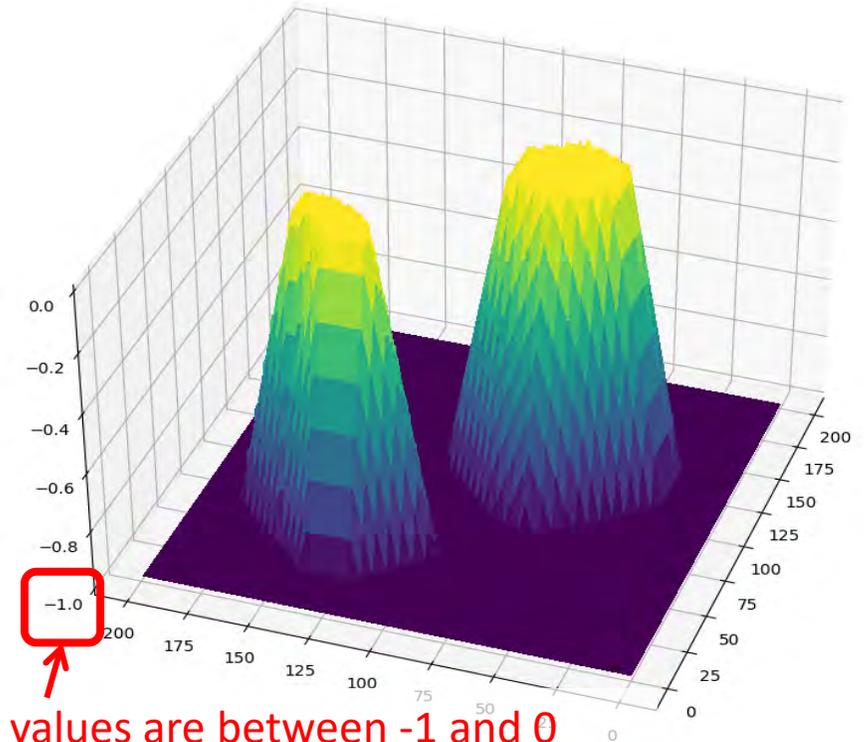
```
repul_field[i] = -min(obstacle_dists[i], THRESH) / THRESH
```



Binary Image



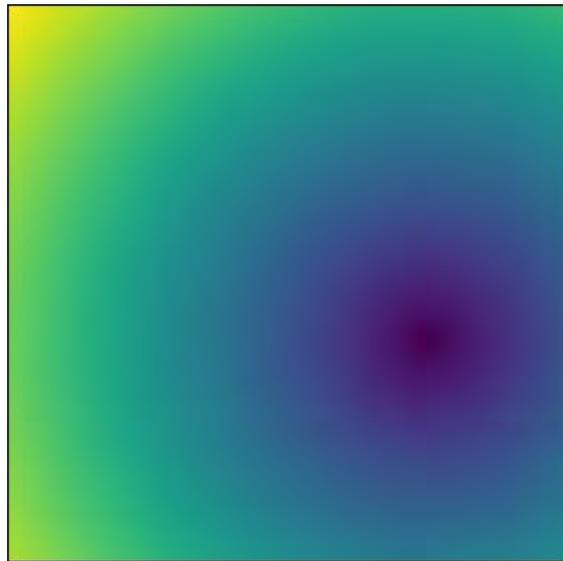
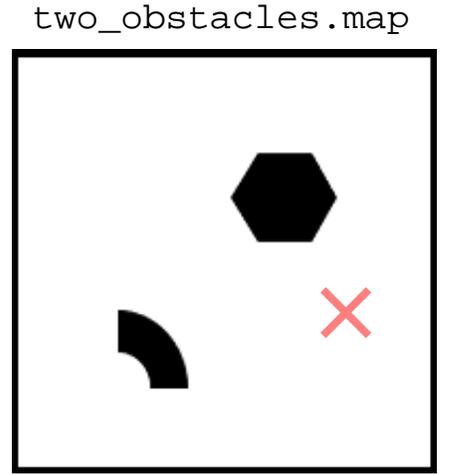
repul_field



All values are between -1 and 0

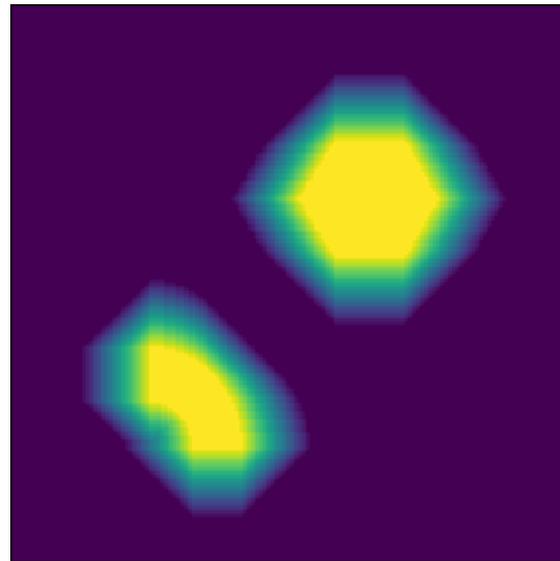
Combining Potentials

Let's combine this new potential with our "cone" attractive potential:



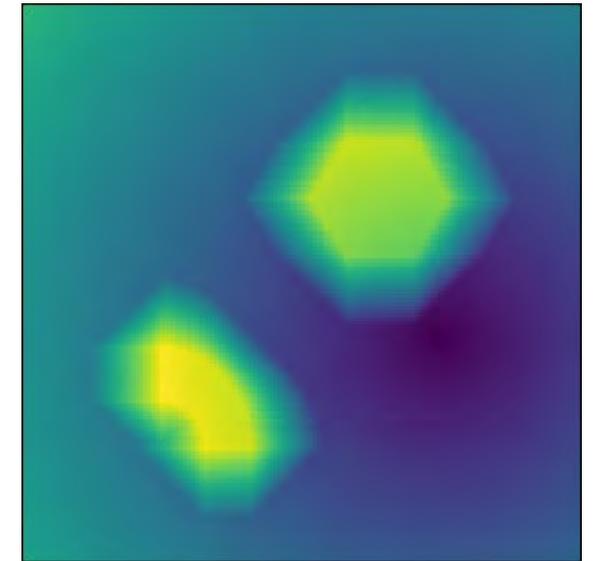
attract_field

+



repul_field

=



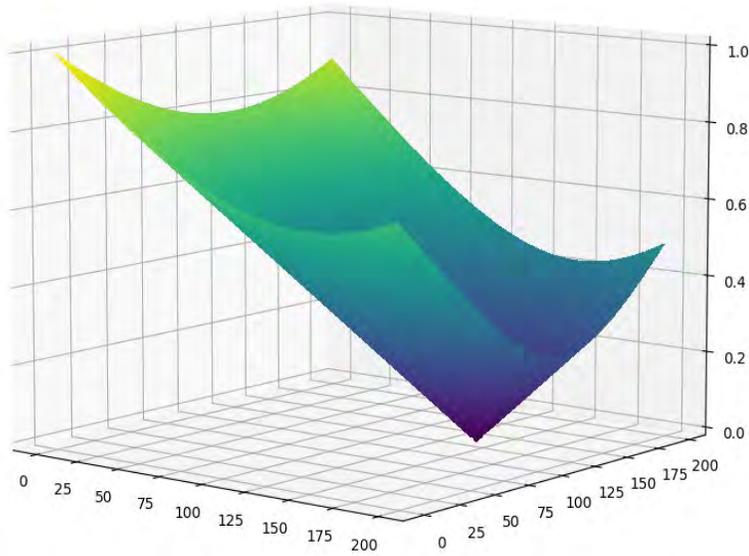
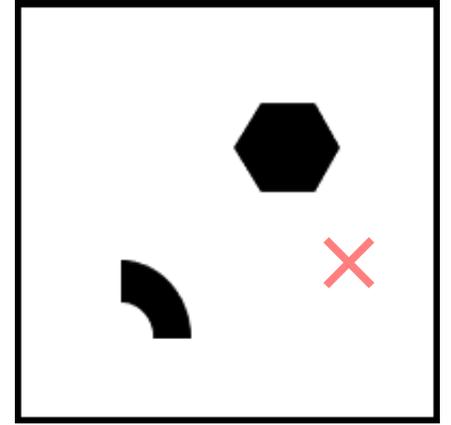
potential

`goal_dists / max(goal_dists)` ← Normalize this too (values from 0 to 1)

Combining Potentials

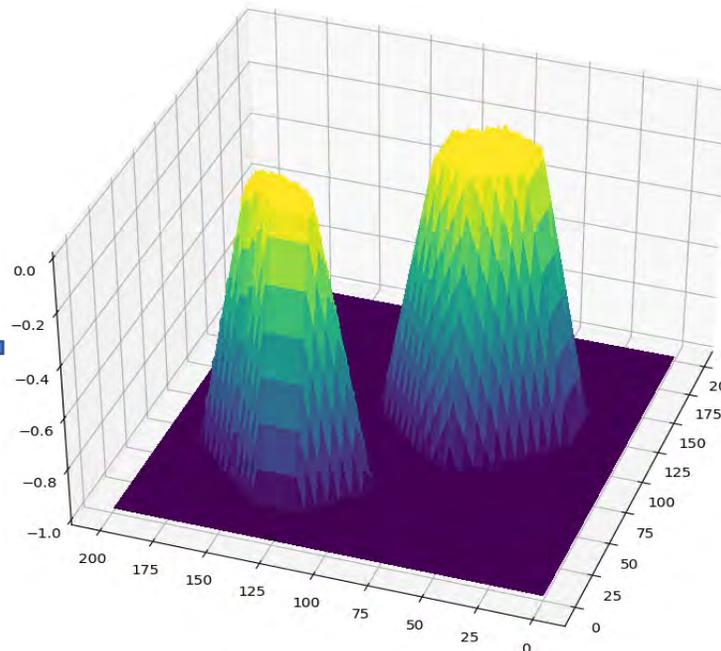
Let's combine this new potential with our "cone" attractive potential:

two_obstacles.map



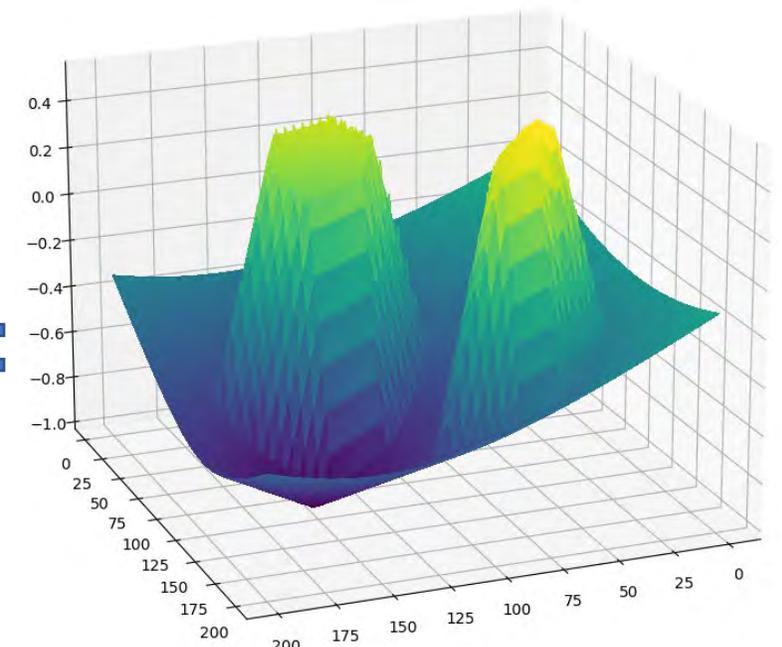
attract_field

+



repul_field

=

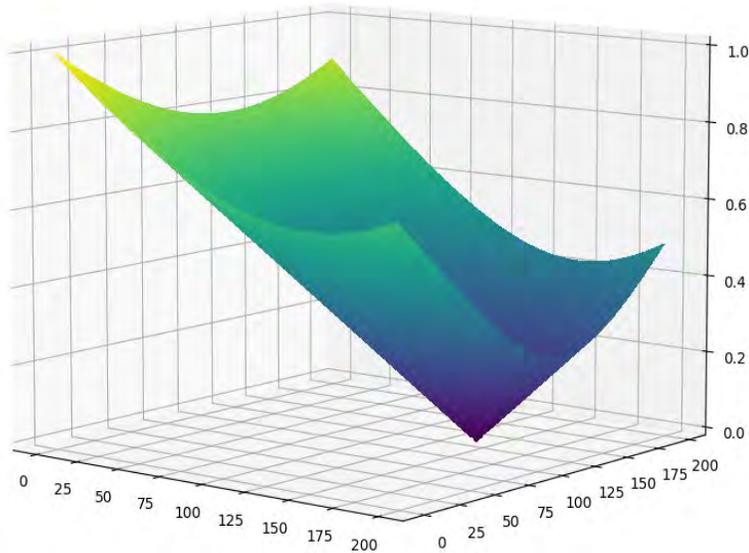
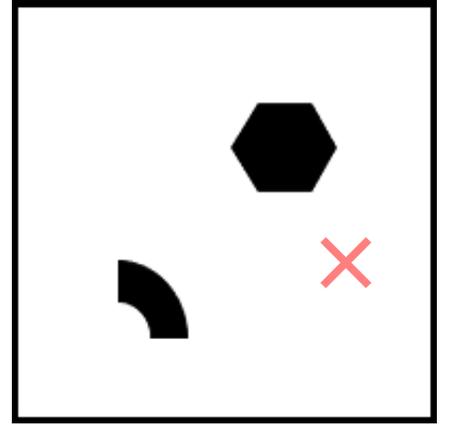


potential

Combining Potentials

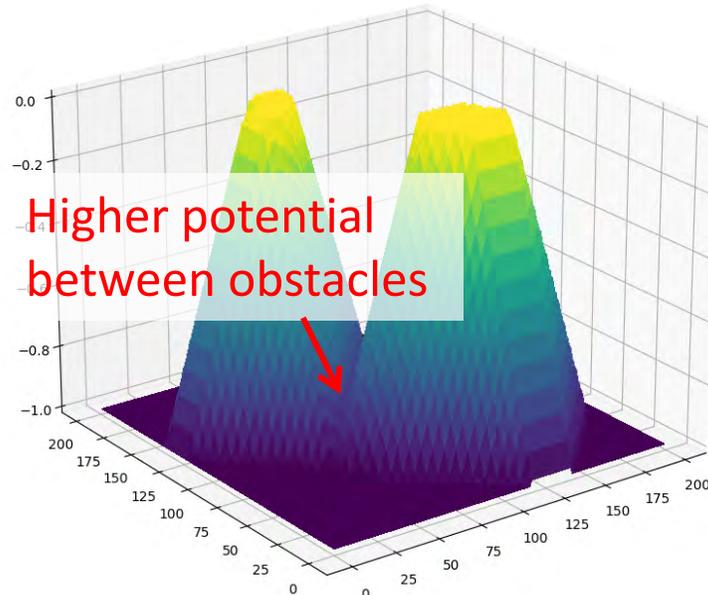
What if we use a **higher** threshold?

two_obstacles.map



attract_field

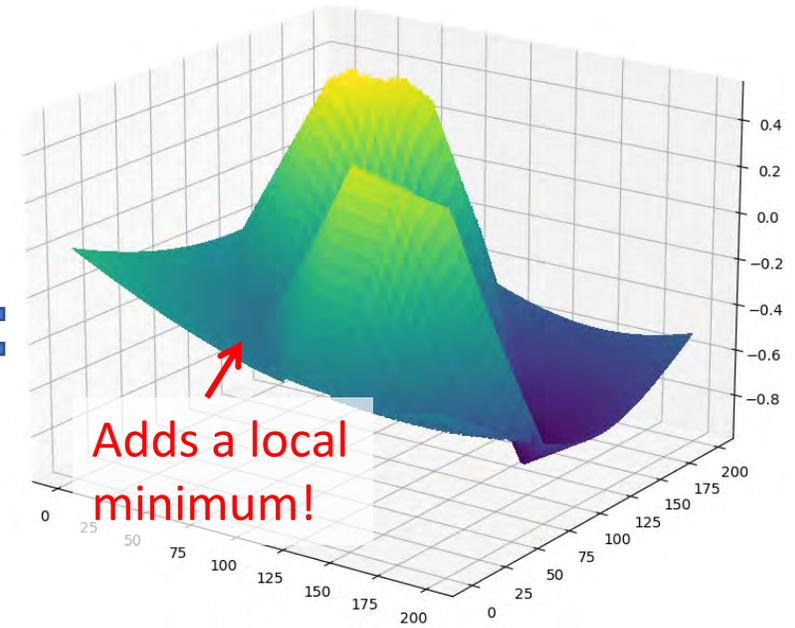
+



Higher potential
between obstacles

repul_field

=

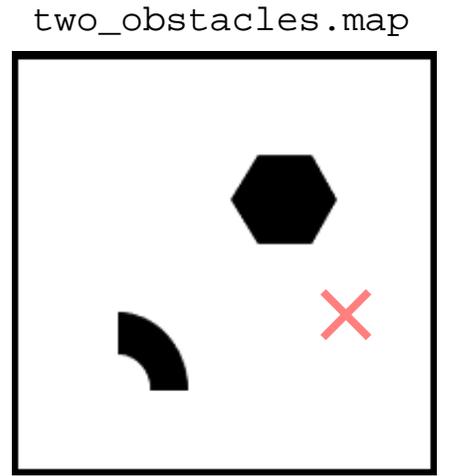


Adds a local
minimum!

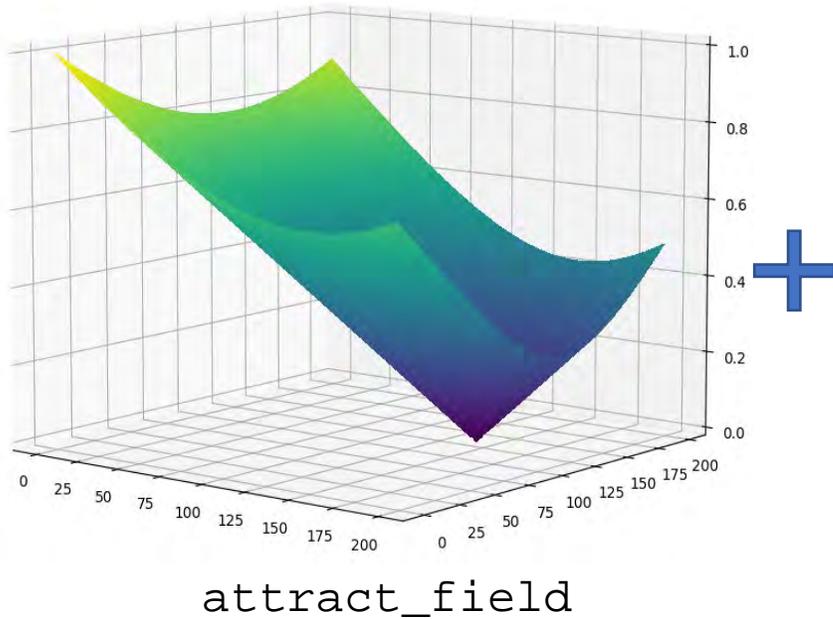
potential

Combining Potentials

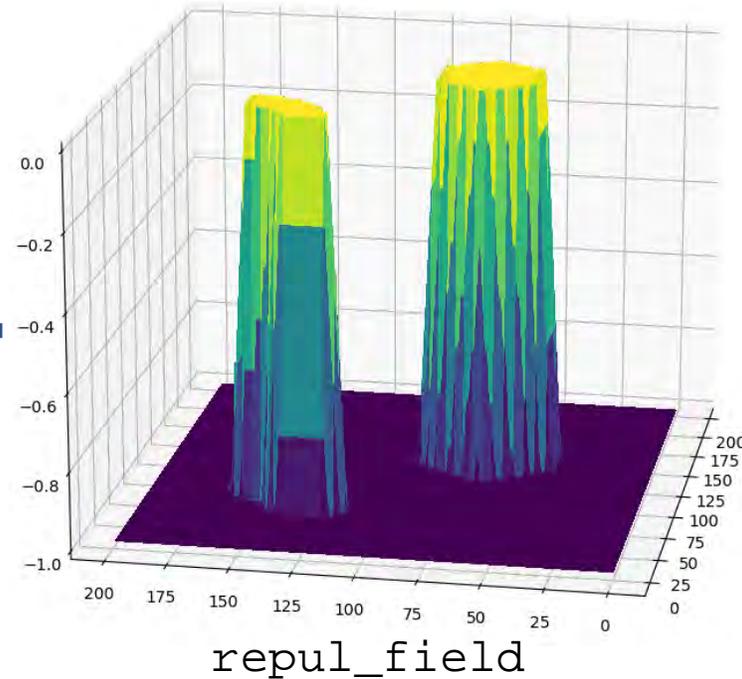
For very small thresholds, the robot might collide with obstacles!



What if we use a **lower** threshold?

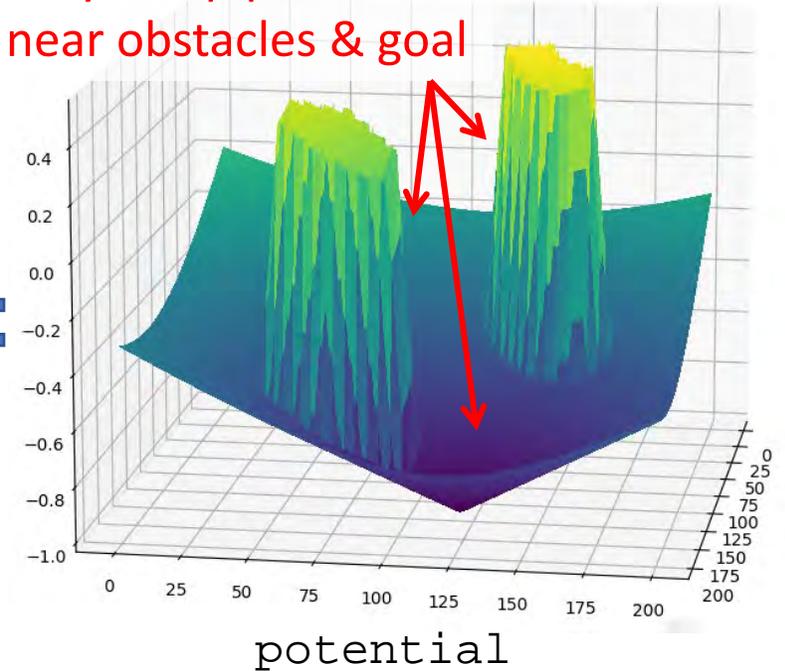


+



=

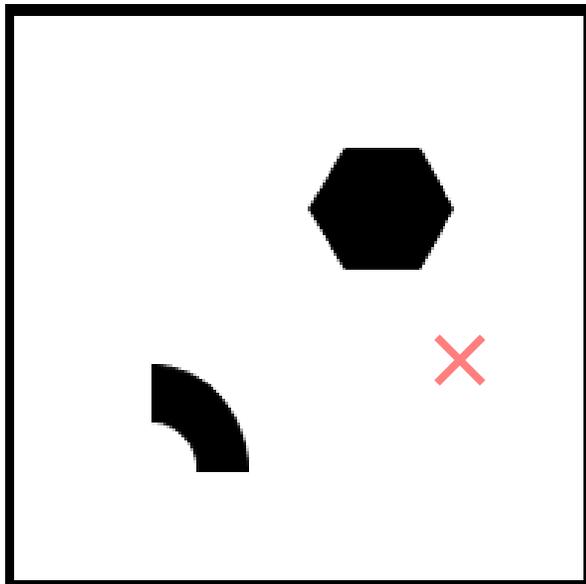
Very steep potential near obstacles & goal



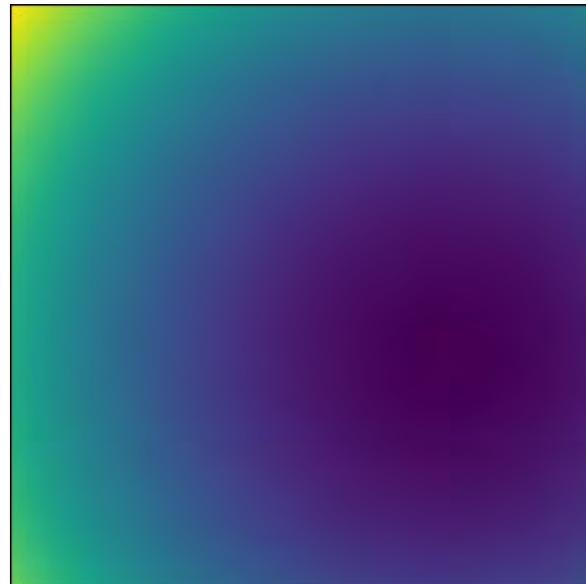
Another Attraction Potential

Idea: Make the slope of the potential steeper far away from the goal and less steep close to the goal.

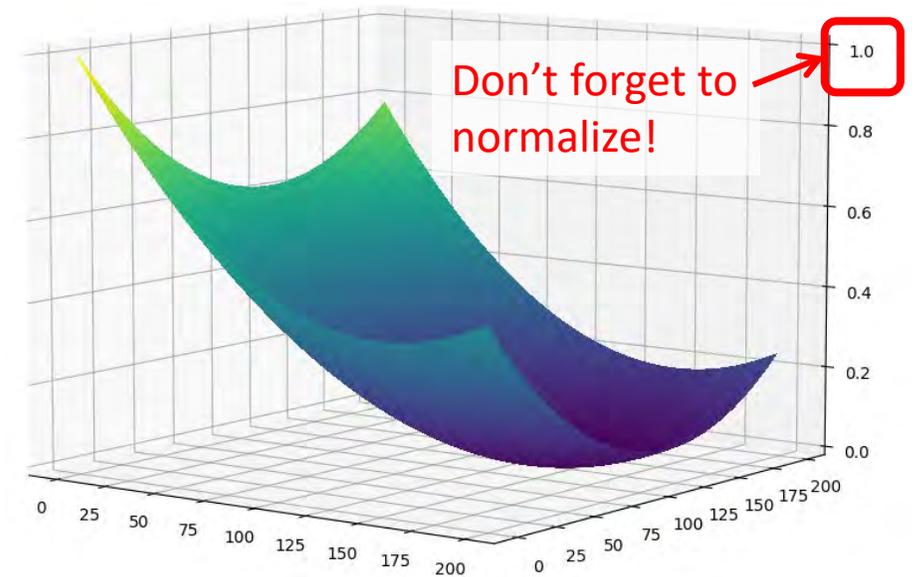
```
attract_field[i] = goal_dists[i] * goal_dists[i]
```



Binary Image



attract_field



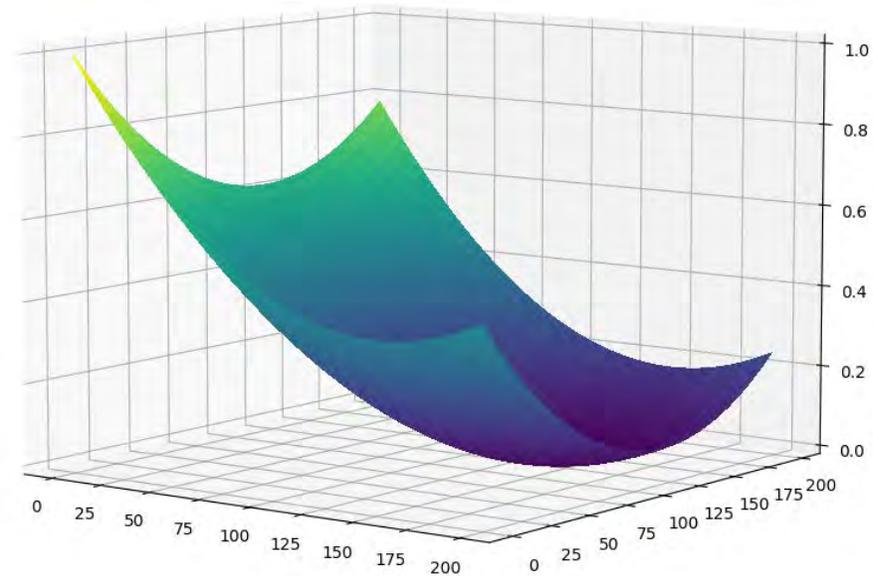
Another Attraction Potential

Idea: Make the slope of the potential steeper far away from the goal and less steep close to the goal.

```
attract_field[i] = goal_dists[i] * goal_dists[i]
```



This defines a **bowl potential**.

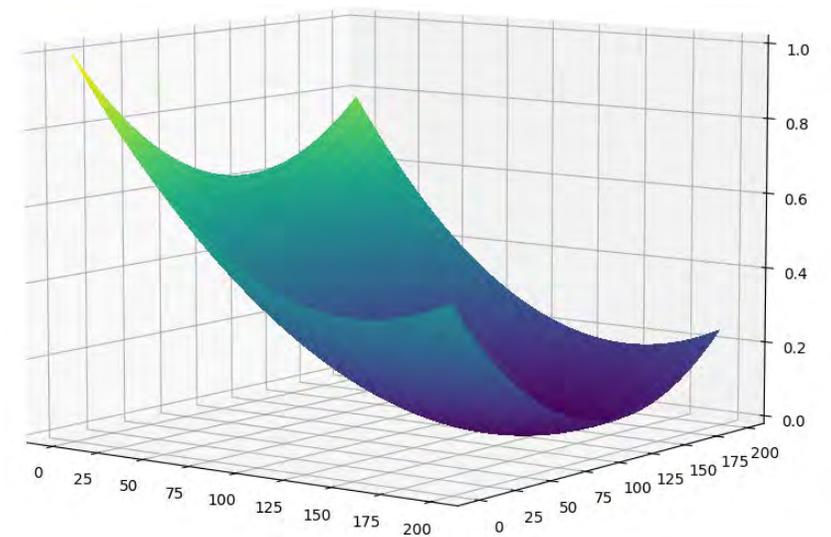


P2.1: Implementing the Attraction Potential

Implement your own version of the attraction potential in the function `createAttractiveField()`. Try different functions!

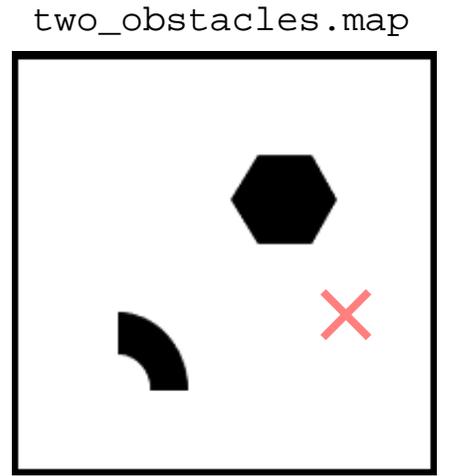
```
33  std::vector<float> createAttractiveField(GridGraph& graph, const Cell& goal)
34  {
35      std::vector<float> attractive_field(graph.width * graph.height, HIGH);
36
37      /**
38       * TODO (P2): Using the graph and the given goal, create an attractive field
39       * which pulls the robot towards the goal. It should be HIGH when far away
40       * from the goal, and LOW when close to the goal.
41       *
42       * Store the result in the vector attractive_field, which should be indexed
43       * the same way as the graph cell data.
44       */
45
46      return attractive_field;
47  }
```

[src/potential_field/potential_field.cpp](#)

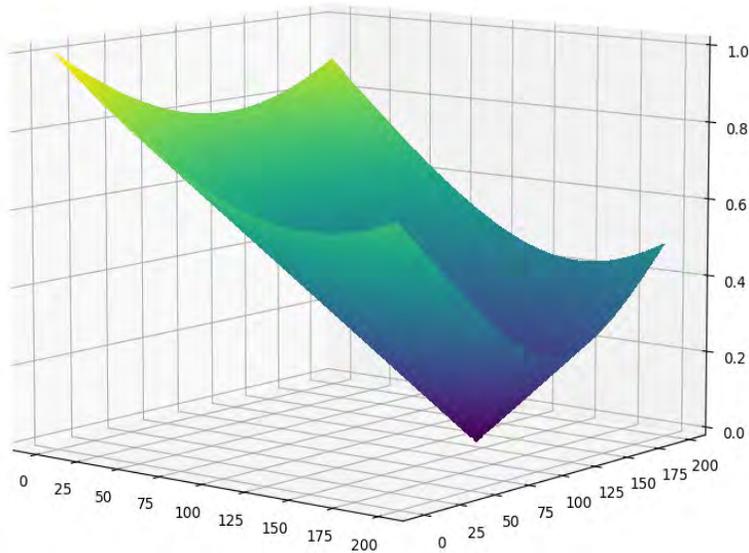


Combining Potentials

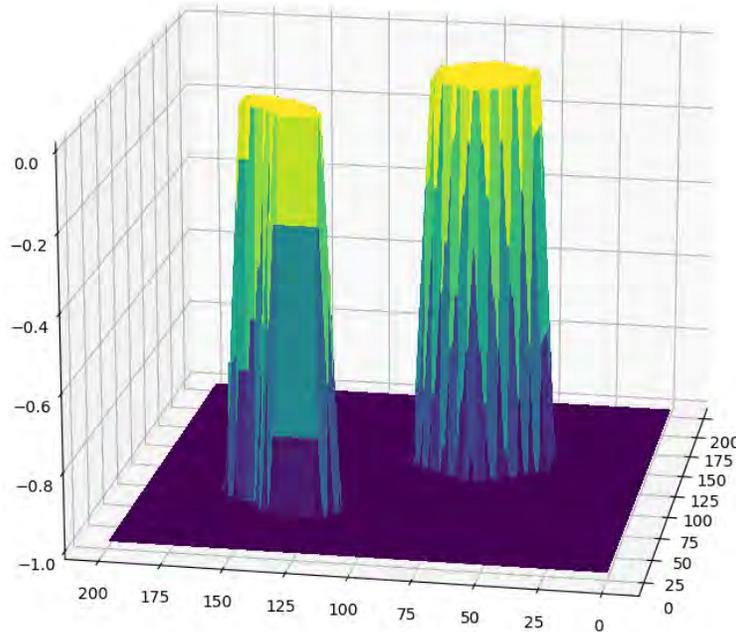
For very small thresholds, the robot might collide with obstacles!



What if we use a **lower** threshold?



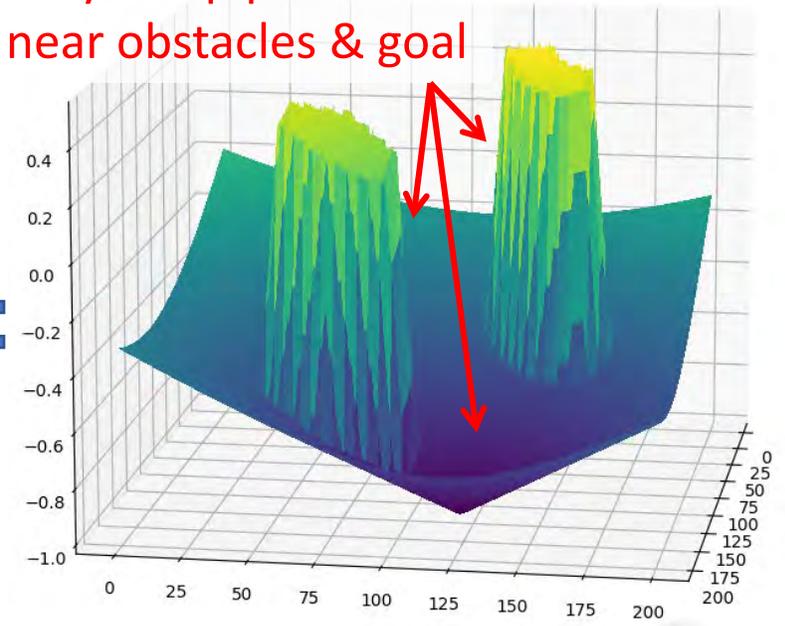
attract_field



repul_field



Very steep potential near obstacles & goal



potential

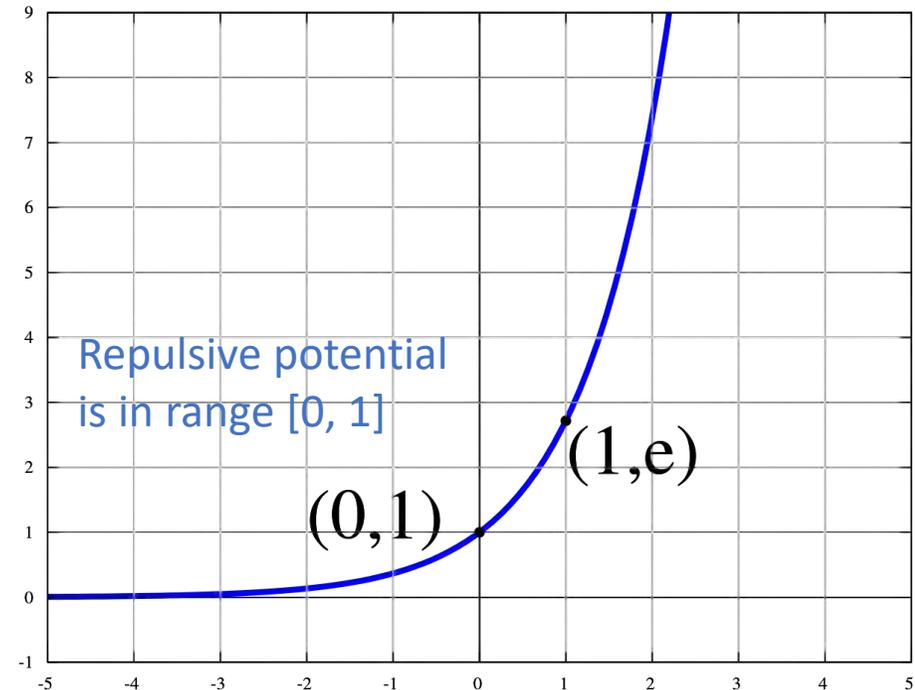
The Exponential Function

The **exponential function** can squash (negative) values between 0 and 1. The slope of the field is steeper closer to obstacles and lower far away from obstacles.

```
repul_field[i] = exp(-obstacle_dists[i])
```

 The exponential function: e^x
 e is Euler's number.

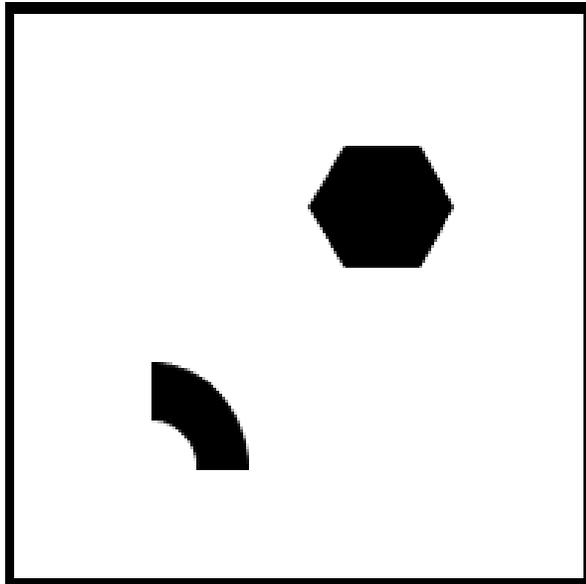
Negative distances are
in range $[-\infty, 0]$



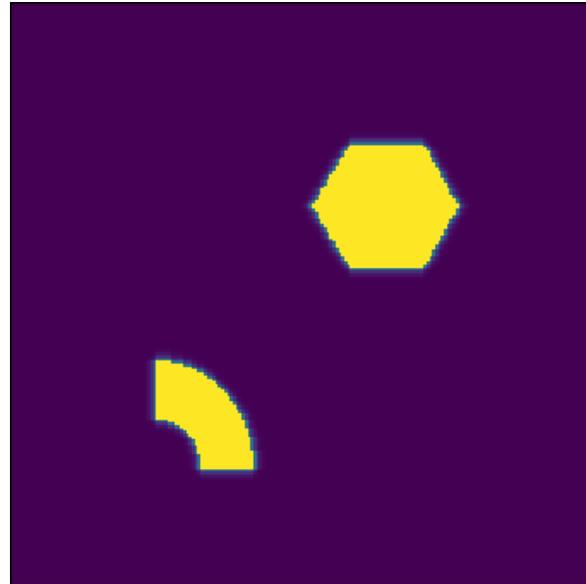
Another Repulsion Potential

Idea: Apply the exponential function to the negative of the distance transform.

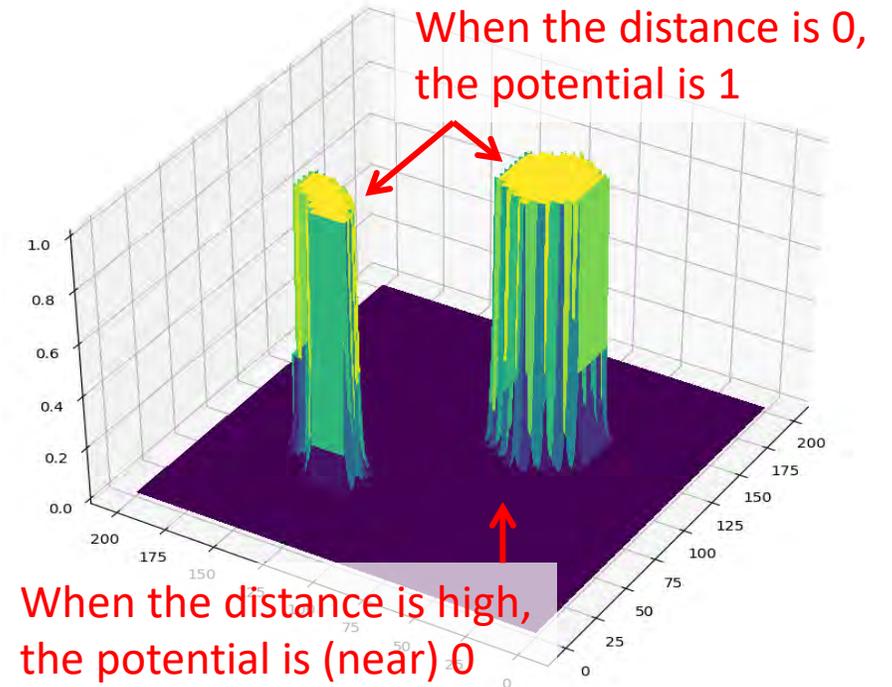
$$\text{repul_field}[i] = \exp(-\text{obstacle_dists}[i])$$



Binary Image

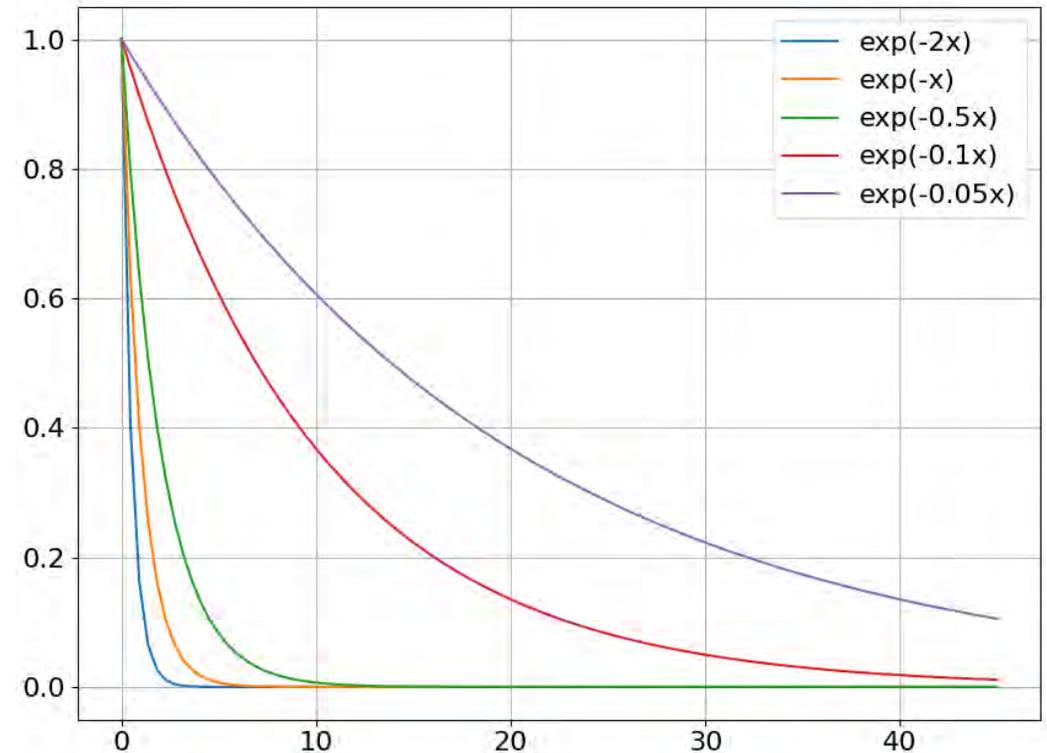


repul_field



The Exponential Function

We can control the steepness of the potential by multiplying the distances by a **coefficient**.



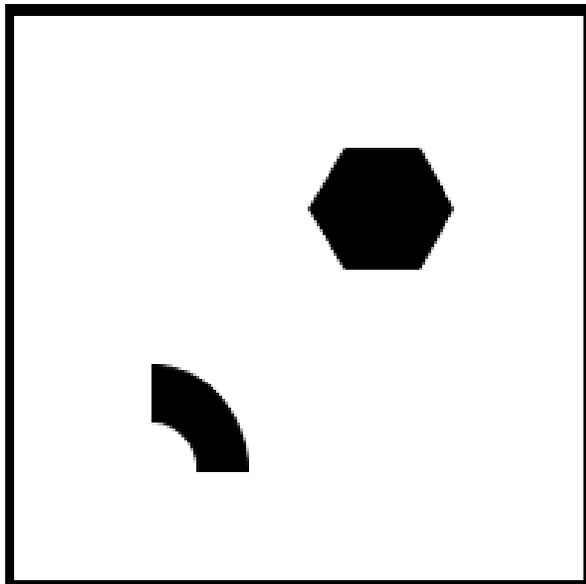
coefficient

$$\text{repul_field}[i] = \exp(-C * \text{obstacle_dists}[i])$$

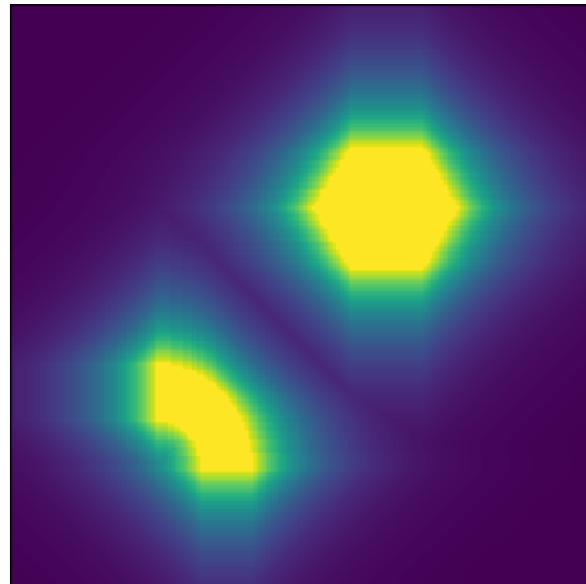
Another Repulsion Potential

Idea: Apply the exponential function to the negative of the distance transform *multiplied by a coefficient*.

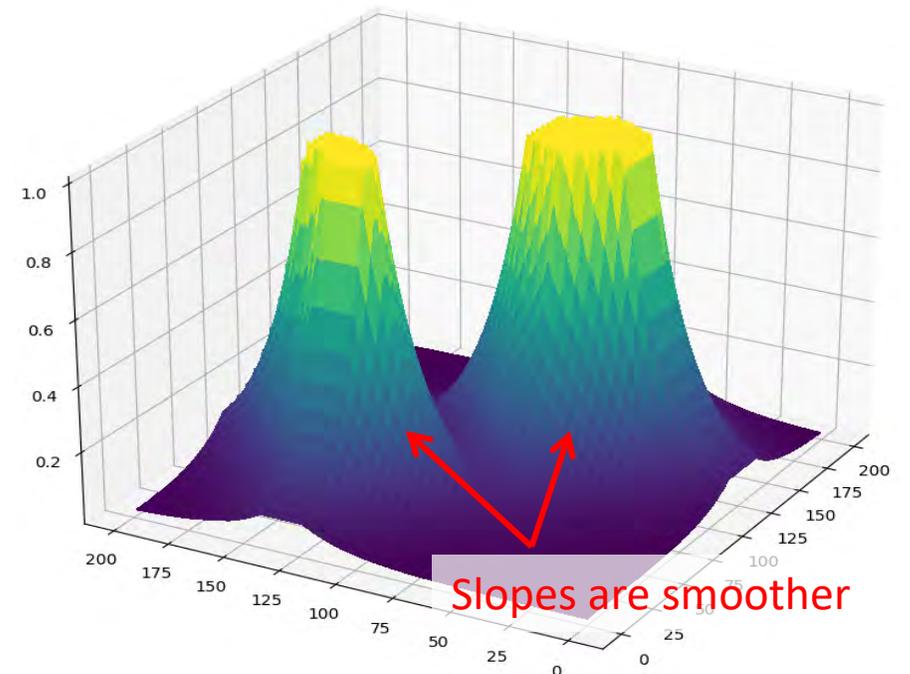
$$\text{repul_field}[i] = \exp(-C * \text{obstacle_dists}[i])$$



Binary Image



repul_field



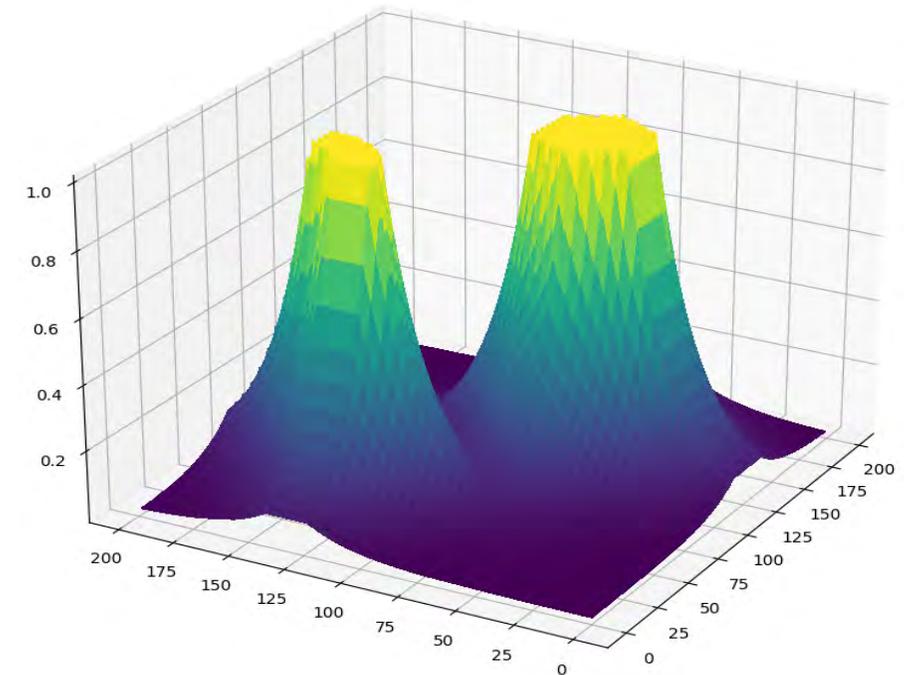
Slopes are smoother

P2.3: Implementing the Repulsion Potential

Implement your own version of the repulsion potential in the function `createRepulsiveField()`. Try different functions!

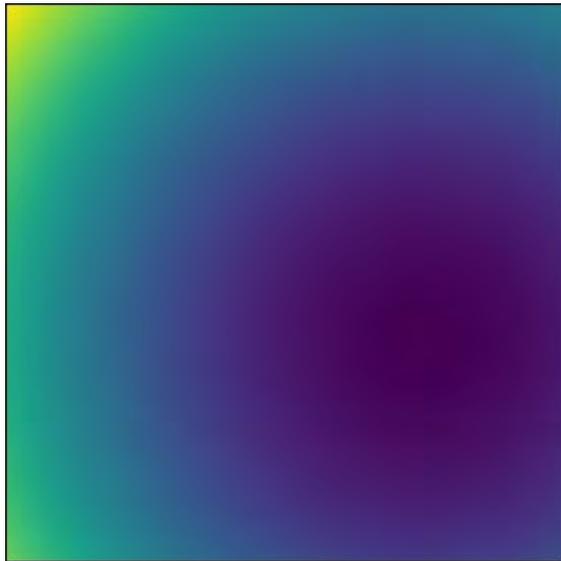
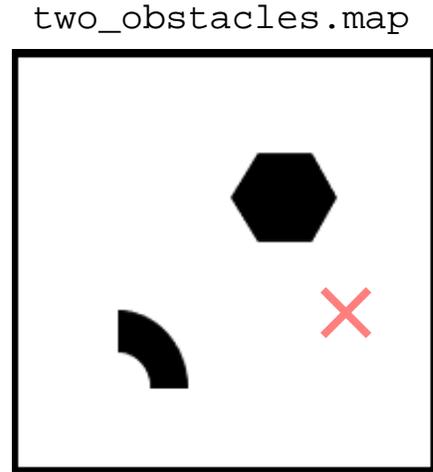
```
50  std::vector<float> createRepulsiveField(GridGraph& graph)
51  {
52      std::vector<float> repulsive_field(graph.width * graph.height, 0);
53
54      /**
55       * TODO (P2): Using the distance transform stored in graph.obstacle_distances,
56       * create a repulsive field which pushes the robot away from obstacles. It
57       * should be HIGH when close to obstacles, and LOW when far from obstacles.
58       *
59       * Store the result in the vector repulsive_field, which should be indexed
60       * the same way as the graph cell data.
61       */
62
63      return repulsive_field;
64  }
```

[src/potential_field/potential_field.cpp](#)



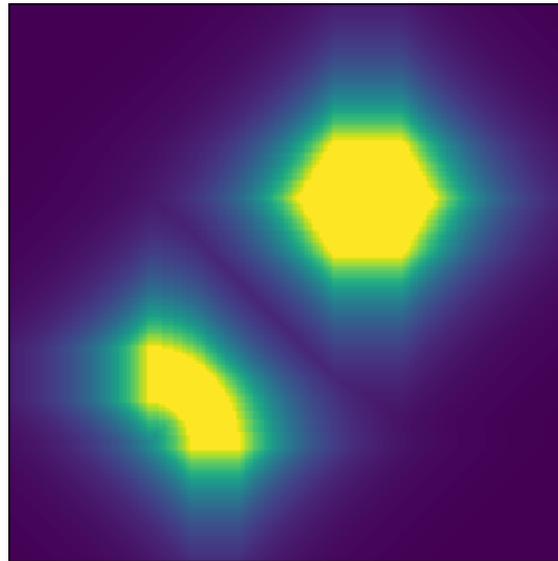
Combining Potentials

Let's combine the exponential repulsion potential with our "bowl" attraction potential:



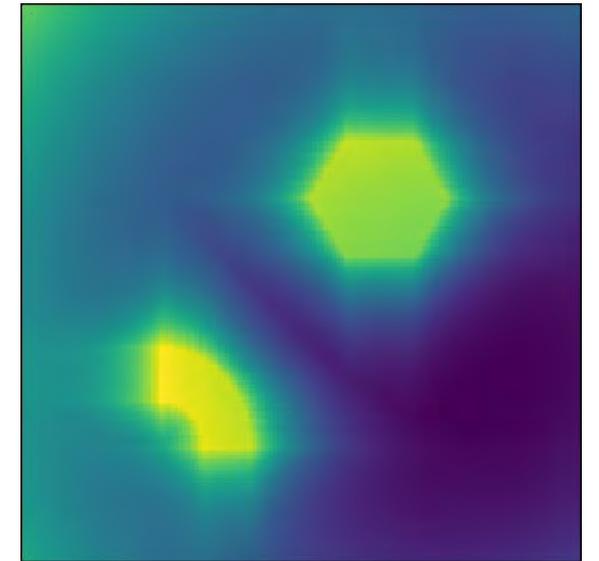
attract_field

+



repul_field

=

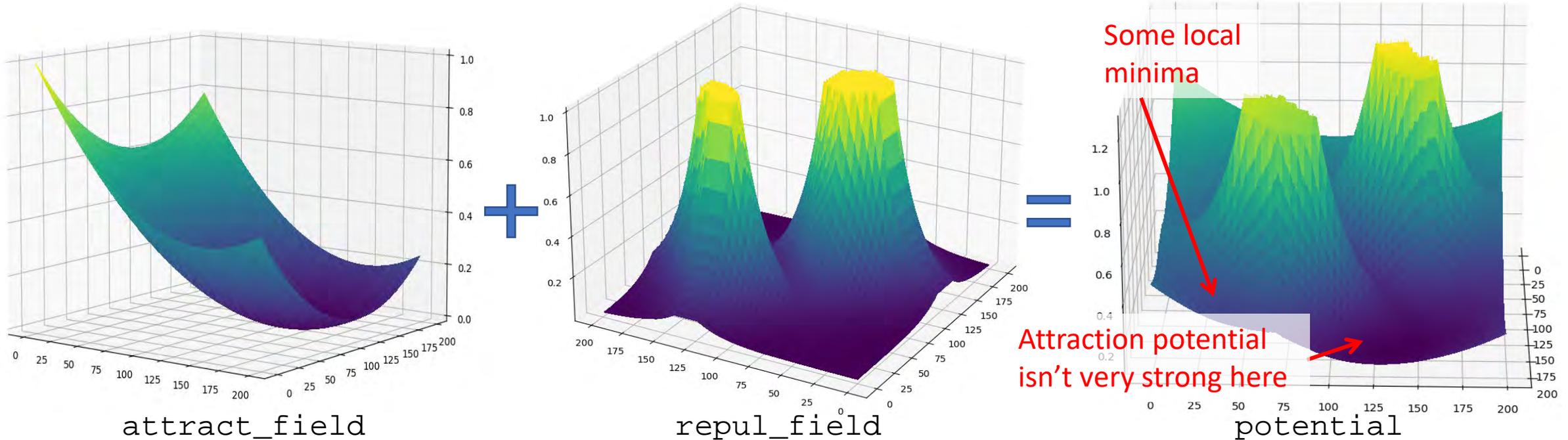
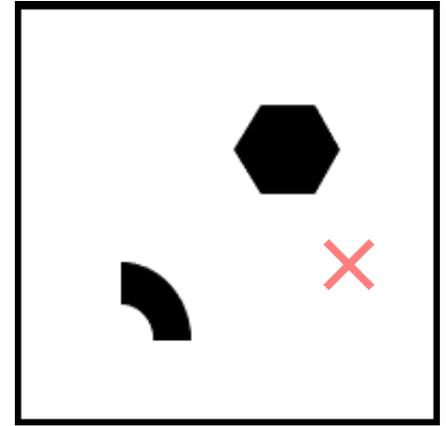


potential

Combining Potentials

Let's combine the exponential repulsion potential with our "bowl" attraction potential:

two_obstacles.map

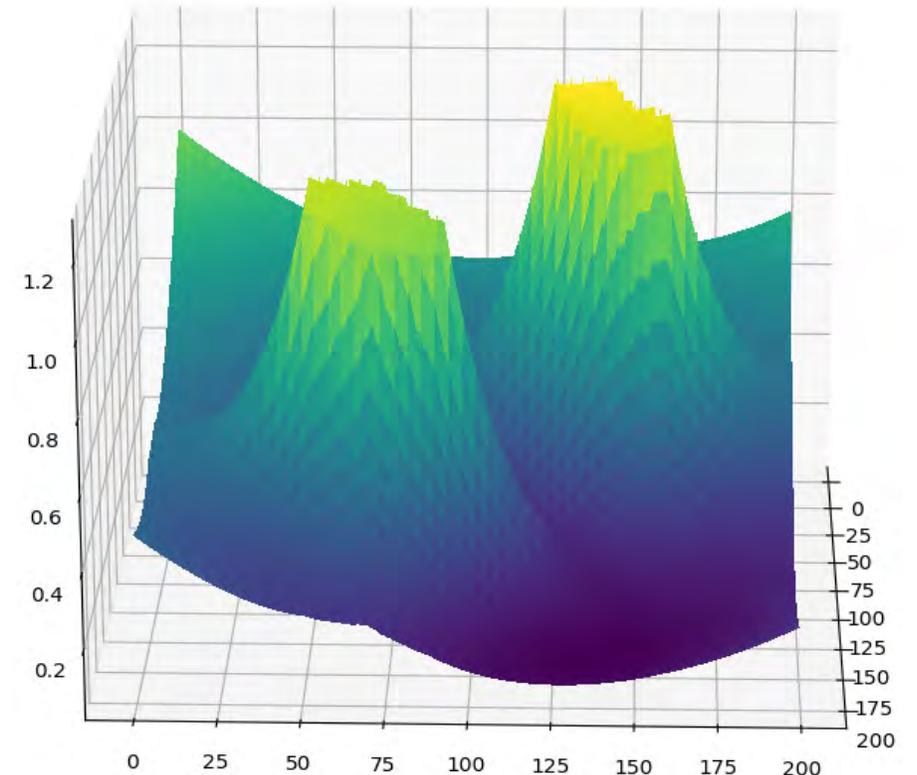


P2.3: Combining Potentials

Combine the attractive and repulsive potentials in function `createPotentialField()`.

```
10  std::vector<float> createPotentialField(GridGraph& graph, const Cell& goal)
11  {
12      std::vector<float> potential_field(graph.width * graph.height, 0);
13
14      /**
15       * TODO (P2): Using the graph and the given goal, create a potential field
16       * which is HIGH in areas the robot should avoid and LOW where the robot
17       * wants to go.
18       *
19       * Store the result in the vector potential_field, which should be indexed
20       * the same way as the graph cell data.
21       *
22       * HINT: The potential field should be a combination of an attractive field
23       * given by createAttractiveField() and a repulsive field created by
24       * createRepulsiveField().
25       *
26       * HINT: Start by using only an attractive field and build from there!
27       */
28
29      return potential_field;
30  }
```

[src/potential_field/potential_field.cpp](#)



Potential Field Navigation

Once we have defined a potential function, we can perform **local search** from any start location.

Local search will navigate to the nearest **local minimum**.

Choose File two_obstacles.map Algorithm Potential Field

Upload Map Clear Goal Plan!

Show Field: Connected

```
repul_field = -obstacle_dists  
attract_field = goal_dists
```

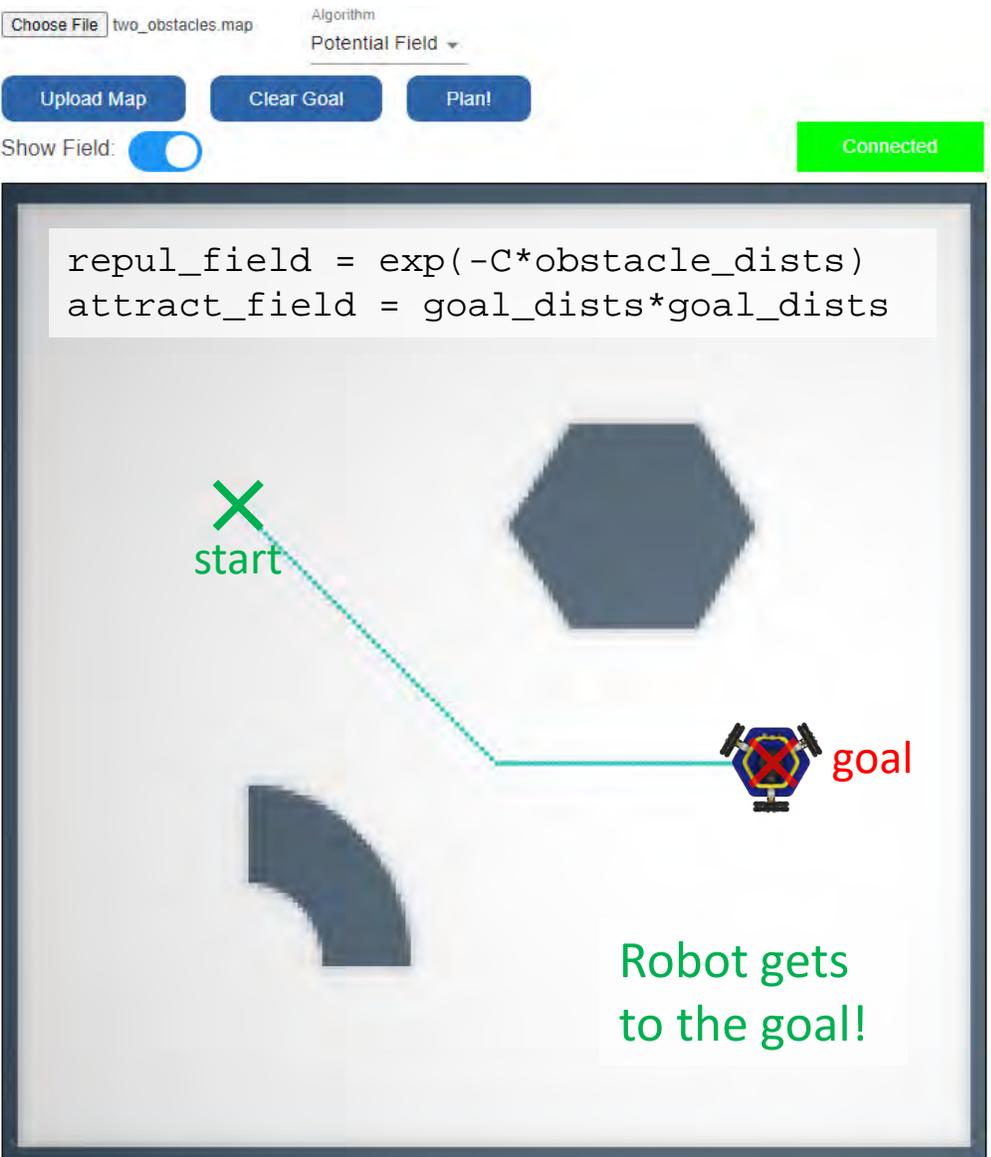
Robot falls into a local minimum

The image shows a 2D navigation environment. At the top, there is a control panel with a file selection button labeled 'two_obstacles.map', an algorithm dropdown menu set to 'Potential Field', and three buttons: 'Upload Map', 'Clear Goal', and 'Plan!'. Below the buttons is a 'Show Field' toggle switch which is turned on, and a green 'Connected' status indicator. The main area is a gray square representing the environment. It contains a robot icon (a blue hexagon with a yellow center) positioned near a red 'X' labeled 'goal'. A green 'X' labeled 'start' is located in the upper left. A green dotted line path starts at the 'start' point, goes up and right, then down and right, ending at the robot. The path is blocked by a large blue hexagonal obstacle in the upper right and a blue quarter-circle obstacle in the lower left. A text box at the bottom right of the environment contains the text 'Robot falls into a local minimum'.

Potential Field Navigation

Once we have defined a potential function, we can perform **local search** from any start location.

Local search will navigate to the nearest **local minimum**.



The screenshot shows a web-based interface for Potential Field Navigation. At the top, there is a "Choose File" button with the filename "two_obstacles.map" and a dropdown menu for "Algorithm" set to "Potential Field". Below these are three buttons: "Upload Map", "Clear Goal", and "Plan!". A "Show Field" toggle switch is turned on, and a green "Connected" status indicator is visible in the top right corner.

The main area displays a 2D environment with a robot icon (a blue hexagon with a red 'X') and a goal icon (a red 'X'). A green path is shown starting from the robot and ending at the goal. The environment contains two obstacles: a dark blue hexagon and a dark blue quarter-circle. A text box at the top of the environment contains the following code:

```
repu_l_fie_l_d = exp(-C*obstacle_dists)
attract_fie_l_d = goal_dists*goal_dists
```

Text labels "start" and "goal" are placed near their respective icons. A green text box at the bottom right of the environment says "Robot gets to the goal!".

Tuning the Potential Field

Choosing a good potential function is an art! Try your algorithm in different maps with different start and goal locations.

Your potential function must generalize as much as possible to different maps, start positions, and end goals! On demo day, you will not be able to tune your functions to a specific map.

Warning: If your distances are in cells, note that the cells aren't always the same size in all maps! (The cell size is stored in `graph.meters_per_cell`)

Potential Field Navigation

Once we have defined a potential function, we can perform **local search** from any start location.

Local search will navigate to the nearest **local minimum**.

The navigation web app is helpful for designing your potential functions.

Choose File two_obstacles.map Algorithm Potential Field

Upload Map Clear Goal Plan!

Show Field: Connected

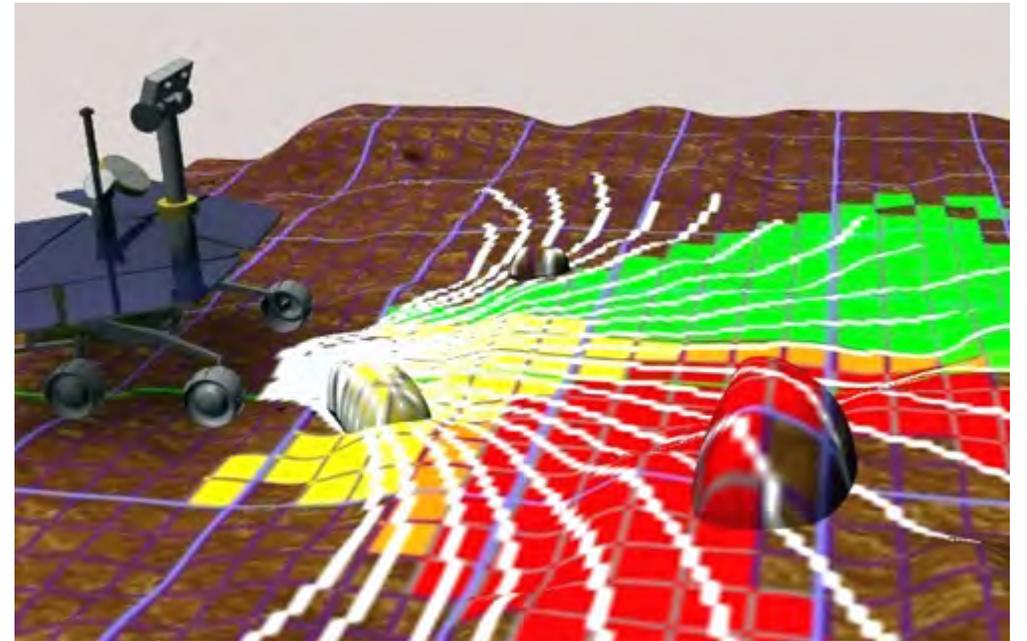
```
repul_field = exp(-C*obstacle_dists)
attract_field = goal_dists*goal_dists
```

Robot still falls into local minima for some start and goal locations

Summary: Potential Field Navigation

Potential field control is used for **local navigation**, to nearby goals.

Can we use potential field navigation for **global navigation**?



Completeness

Is there a potential field that will allow the robot to navigate to the goal in this map?

A navigation algorithm is **complete** if it always finds a path when one exists, or detects when no path exists.

Project 3: Graph search algorithms for robot navigation.



Project 2: Potential Field Navigation

- ☑ Build a map of environment
- ☑ Form attraction potential to goal
- ☑ Form repulsion potentials away from obstacles
- ☑ Add potentials together into potential field
- ☑ Local search over potential field to navigate

← Today!

